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A dynamic model for airframe cost estimation

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A DYNAMIC MODEL FOR AIRFRAME
COST ESTIMATION

by

Ronald Lee Brown

March 1986

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A Dynamic Model for Airframe Cost Estimation

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requirements for the degree of

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ABSTRACT

The Department of Defense has historically favored a relatively simple parametric approach to cost estimation. Economic theory has largely been ignored and the learning curve has become the customary analytical tool for relating production quantities to airframe costs. This research examines an effort to synthesize neoclassical economic theory with the traditional learning curve methodology. The proposed model implements a dynamic cost function that considers the effects of learning and production rate on the production process. To empirically test its validity, the model is applied to the F-4 Phantom II production program and parameters are estimated using historical production data.

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I. INTRODUCTION

As we are reminded on an all too frequent basis, weapons systems acquisitions by the Department of Defense (DoD) have been plagued by a series of cost overruns. With the current political climate of intense scrutiny of government expenditures in an effort to shrink the federal budget and battle record deficits, the DoD can ill afford any such adverse publicity. Some of the implications of continued deficient cost estimation are particularly disturbing. Using pessimistic cost estimates that encompass all uncertainty will minimize the chance of a cost overun by providing cost estimates that are consistently too high. Of real concern is the possibility that these unnecessarily high estimates might become self-fulfilling prophecies. Such cost estimates might also prompt a decision to invest in an alternative system that is less costly but also less cost effective--the end result would be an inefficient allocation of valuable economic resources. At the other extreme, a cost estimate that is too low can lead to insufficient funding which, if not supplemented, could delay deliveries of needed weapons or prematurely terminate production at output quantities that are less than optimal. Another effect, with possibly more serious consequences, is the possible erosion of public trust in our military forces if the public perceives mismanagement and waste are perceived to be the source of cost overruns.

Some of our cost estimation difficulties stem from design changes or improvements that occur after a cost analyst has formulated an estimate of a program's cost. Such problems are beyond the analyst's control; unfortunately, cost estimation problems have occurred in many programs that didn't experience such changes. A possible conclusion is that our current cost estimation methodology is inadequate and that a better understanding of the factors that determine cost is required.

Current methods of cost estimation can be divided into two approaches: (1) the "topdown" or statistical approach which generates simple, imprecise estimates based on existing similar systems and which is insensitive to many production decisions and (2) the "bottom up" or industrial engineering approach which examines separate segments of work in great detail and synthesizes these estimates into a complex, yet often imprecise, total cost estimate which requires substantial revision when changes occur. Neither of these approaches is particularly helpful to a program manager who must develop a funding profile suitable for projected lot release dates and delivery schedules and then deal with subsequent changes in funding or production schedule.

This paper concentrates on a model developed principally by Gulledge and Womer [Ref. 1] that strives to model the factors influencing the cost of airframe production. In particular, the influences of production rate and delivery schedule, as well as the standard learning effects, are thought to significantly affect program costs. These factors, together with basic economic theory, are incorporated into a model which aspires to encourage wiser acquisition policy by providing greater sensitivity to alternative policy decisions. The chapters that follow will provide a historical perspective of cost estimation methodology, review the proposed model, evaluate its performance using data on F-4B airframes purchased by the Navy, and investigate the model's applications.

II. HISTORICAL PERSPECTIVE

The DoD has historically favored a relatively simple parametric approach to cost estimation. Costs are modeled as a function of only a few aircraft design and performance characteristics and unit costs are expected to decrease as learning accumulates with production experience. The model considered in this paper is unique in that it attempts to model the effects of production rates as well as learning and, implicitly, the effects of facility size on total program costs. Thus, it is an integration of what have generally been two distinct approaches to cost estimation: (1) the neoclassical economic approach which purports production rate to be a significant determinant of cost and (2) engineering cost studies which rely heavily on the learning curve to relate costs to the number of items produced but not to the rate at which they are produced.

Economic theory typically depicts the relationship between unit production costs and output per production period as a U-shaped function. Such a shape indicates that, as the level of output increases, unit costs fall over a certain range and then begin to rise. This reflects the powerful logic of the law of diminishing returns which, as explained by Samuelson [Ref. 2], says

> An increase in some inputs relative to other fixed inputs will, in a given state of technology, cause total output to increase; but after a point the extra output resulting from the same additions of extra inputs is likely to become less and less. This falling off of extra returns is a consequence of the fact that new "doses" of the varying resources have less and less of the fixed resources to work with.

Johnston [Ref. 3] cites the concept of diminishing marginal product as one of the most common generalizations of the law of diminishing returns. Diminishing marginal product refers to the eventual decreasing increments in output that occur when the quantity of an input is increased by equal increments, with all other input quantities remaining fixed. A tendency for varying factors to show diminishing returns when applied to fixed factors implies a tendency for marginal cost to be rising. If at first there are increasing returns, there are also declining marginal costs--but ultimately diminishing returns and increasing marginal cost will prevail. Such a formulation is consistent with the notion that, for a firm operating in perfectly competitive product and factor markets, marginal cost at equilibrium must be rising.

The implication of diminishing returns for an airframe cost model is reasonably clear. An increase in production rate will, at least in the short term, require additional labor. Since the production facility itself must be considered fixed, consider two alternatives proposed by Washburn [Ref. 4] that could be used to increase the production rate: (1) overtime or weekend work where a premium is paid for labor at odd hours; or (2) hiring additional manpower at a loss of efficiency due to crowding, scheduling, etc. Economic theory thus leads us to the conclusion that production rate and production costs are positively related; that is, with higher production rates, labor is more costly and/or less efficient, and, as a consequence, costs per unit of output increase. While firmly grounded in economic theory, this conclusion has suffered from a lack of empirical verification.

In contrast to the neoclassical approach is the engineering cost study technique which incorporates the progress function or learning curve concept developed by Wright [Ref. 5]. In his pioneering analysis of aircraft production costs, Wright noted that labor hours per unit of output decreased as cumulative output increased and proposed the function $F = N^X$ (F is a factor of cost variation proportional to N, the quantity produced; X is a learning parameter) to explain the labor cost-production quantity relationship. Wright's data were consistent with the eighty percent learning curve which has received widespread acceptance as an industry "standard". As explained by Wright, greater production improves worker efficiency and makes additional tooling economically feasible so that a firm incurs only an eighty percent increase in labor costs when output is doubled. It should be noted that Wright's research and the factors derived assumed that no major changes would be introduced during production.

It is readily apparent that production rate and production quantity are positively correlated so it seems that these two approaches to cost estimation are contradictory. The first hypothesizes that unit production costs are directly related to the rate of production while the second hypothesizes that unit production costs are inversely related to the cumulative volume of production which, in turn, is directly related to the rate of production. Further complicating the puzzle is that the first approach is based on solid, well-accepted economic theory with little empirical support while the other is seemingly adopted to explain the data and has little theoretical underpinnings.

Numerous research efforts have been undertaken to investigate the ambiguity of the cost/quantity/rate relationship. In Asher's [Ref. 6] study of cost-quantity relationships in airframe production, he concluded that the conventional learning curve may not be an entirely accurate description of the relationship between unit cost and cumulative output. For certain values of cumulative output, he found that the labor cost curve became convex which would be consistent with the diminishing returns expected by the economist's approach.

In subsequent research, Alchian [Ref. 7] reformulated the traditional cost function to explain much of this apparent contradiction. While acknowledging that all of the characteristics of a production operation can affect its cost, Alchian directs attention to x(t)--the rate of output, V--the total contemplated volume of output, T--the first scheduled delivery date, and m--the time interval between the first and last scheduled deliveries. These four characteristics comprise the input variables for the Alchian cost function, $C = f(V,x,T,m)$. On this function are based Alchian's nine propositions to explain the cost-quantity relationship. All other factors being fixed, Alchian contends that: (1) $\partial C/\partial x(t) > 0$, cost varies directly with production rate; (2) $\partial^2 C/\partial x^2 > 0$, the increment in cost is an increasing function of production rate; (3) $\partial C/\partial V > 0$, cost varies directly with the total anticipated volume of outpUT; (4) $\partial^2 C/\partial V^2 < 0$, the increment in cost is a decreasing function of total volume; (5) $\partial (C/V)/\partial V < 0$, cost per unit decreases as total volume increases; (6) $\partial^2 V/\partial V \partial x < 0$, marginal discounted cost of increased quantity of output decreases as production rate increases; (7) $\partial C/\partial T < 0$, cost varies inversely with the elapsed time until first delivery; (8) all derivatives in propositions 1-5 are diminishing functions of T, but not all diminish at the same rate; and (9) the cost of future output declines as the total quantity produced increases. Propositions 1, 2, 3, 7 can be said to encompass the notion of diminishing returns put forth by the conventional economic approach while propositions 4, 5, 6, 9 support the learning curve notion of decreasing unit costs as total volume increases. Even without the learning effect, Alchian felt that $\partial C/\partial V$ would be decreasing. This second type of cost saving depends on scheduling larger volumes of production in advance which may allow a reduction in capital cost or operating costs per unit.

As further explained by Hirshleifer [Ref. 8], the key idea is that cost can be regarded as a function of the quantity of output in two different dimensions: the rate of output and the scheduled volume of output. The first of these is a flow measure while the second is a stock measure and, as pointed out by Alchian [Ref. 7], the response of cost to changes in the one direction is qualitatively different from the response to changes in the other dimension of output. Thus, marginal cost can always be a rising function of the rate of output, volume held constant, and a falling function of output volume, rate being held constant, and neither of the aforementioned theories of cost estimation is contradicted.

In their research of Alchian's proposals, Preston and Keachie [Ref. 9] contrasted the U or L-shaped curve typically used in economic analysis to depict the unit cost-production rate relationship with the hyperbolic learning curve that models unit costs as a continuously decreasing function of total output. The learning curve approach was discredited for ignoring the effects of production rate. They felt the production rate could not be assumed constant nor could it be considered an immaterial factor of unit costs. Their suggested integration of the economist's cost function and the learning curve involves three variables: C_t, the level of production costs for a production period; q_t, the amount of output per production period; and V, the accumulated level of output. Their cost function can be displayed graphically as

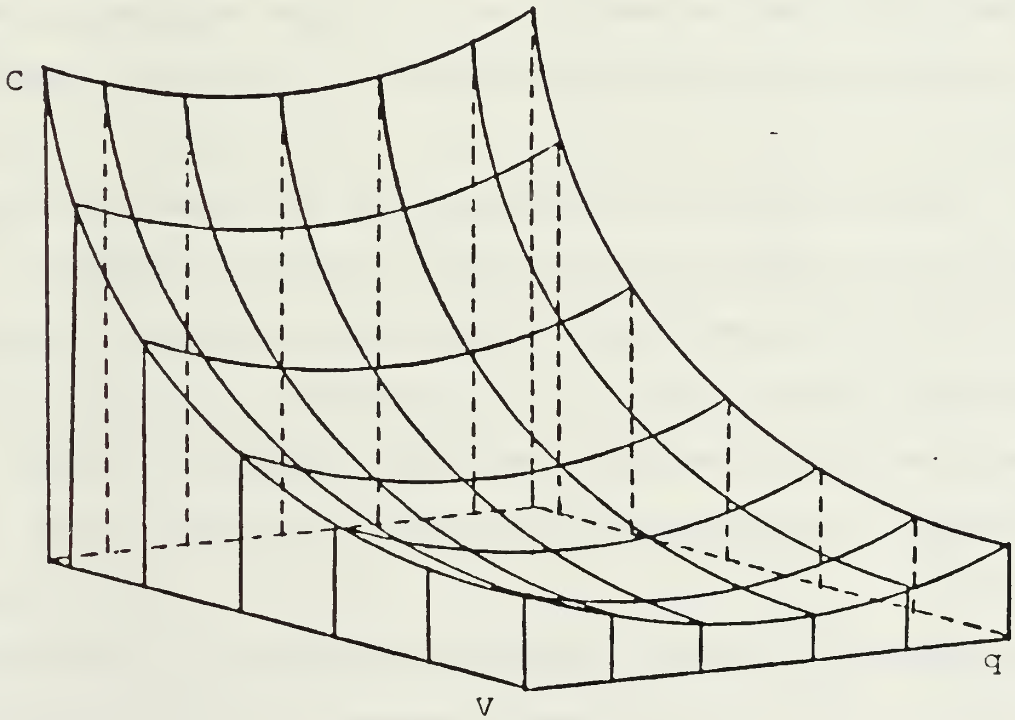


Figure 1. Integrated Cost Function

The U-shaped curve shows the combined effects of the spreading of fixed costs and the gradual rising rate of variable costs, while the downward slope with respect to V shows the learning effect.

Using a similar approach, Cox and Gansler [Ref. 10] noted that a move from a relatively inefficient q_t to one near the optimum results in more rapid cost decreases than would have resulted from a standard cost improvement curve. Conversely, a move from an efficient q_t near the optimum to a less efficient rate some distance from the optimum results in a reduced rate of cost improvement from what we would expect from a standard cost improvement curve. A cost increase could, and would, result if the new production rate is significantly different from the previous, more efficient rate.

As is becoming apparent, the interrelationship of production rate and the total volume of output is quite complicated and difficult to unravel. Alchian's postulates are theoretically sound but empirically unvalidated due to the nature of cost data and the complexity of the relationship. For example, when both volume and rate of output change in the same direction, Alchian's postulates are inconclusive--the two expected effects work in opposite directions and the net result cannot be reliably predicted. It has also been shown that both the magnitude and direction of change in unit cost depend upon the extent of movement toward or away from the optimal production rate when a rate change is required.

Considering the intricacy of this cost-quantity-rate relationship, it is not surprising that several studies have found contradictory results. Large et. al. [Ref. 11] were forced to conclude that the influence of production rate could not be predicted with confidence and could probably be ignored since other uncertainties prevailed. Their analysis found that rate effects were often negligible and sometimes contrary to the a priori notion that production rate and unit costs were inversely related. They also noted that high volume, high production rate programs were not typical of recent DoD aircraft procurements--of the 29 acquisition programs they surveyed, 18 had production runs of less than 500 and only eight had production runs of greater than 1000. This provides empirical evidence that production facilities are typically not built to sustain high production rates and lends credence to Washburn's previously cited contention that raising the production rate requires hiring additional labor at a premium since the production facility itself is fixed.

Research by Smith [Ref. 12] and later by Congleton and Kinton [Ref. 13] used the cost function

$$\text{Cost} = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \tag{1.1}$$

where β_0 = cost of first unit

X_1 = cumulative quantity produced

β_1 = slope of quantity/cost curve

X_2 = rate of production

β_2 = slope of rate/cost curve

and found unit costs to be significantly and negatively correlated with production rate. Bemis [Ref. 14] obtained similar results with his model for examining the effects of production rate on the cost of many defense items. Cox and Gansler's [Ref. 10] study of tactical missile programs found both positive and negative correlation of unit costs with production rate.

Womer [Ref. 15] postulates that analyses such as these that conclude that unit costs decrease as the rate of production increases (i.e., increasing returns to scale) are inconsistent with or ignore optimal contractor behavior. As noted by Washburn [Ref. 4], consistently increasing or even proportional returns to scale would, in the presence of discounting, lead to an optimal production program that crowds all production into an arbitrarily short period at the very end.

In a model which predates the one considered in this thesis, Womer [Ref. 15] incorporates the research of both Alchian [Ref. 7] and Washburn [Ref. 4] into a production function that relates the rate of resource use and cumulative production experience to the output rate while being consistent with logical contractor behavior. The following production function is specified:

$$q(t) = AQ^{\delta}(t)\,x^{1/\gamma}(t) \tag{1.2}$$

where $q(t)$ = output rate at t

$Q(t)$ = cumulative output at t

$x(t)$ = rate of variable resource use at t

δ = learning parameter; $0 \leq \delta \leq 1$

γ = returns to scale parameter; $\gamma > 1$

A = a constant, $A > 0$

By restricting γ to values greater than one, it is assumed that diminishing returns exist for the variable resource which insures that the model makes economic sense. It is this key assumption that distinguishes Womer's model from most others. Solving this production function for $x(t)$ and integrating over time yields the amount of the variable resource used during the time period in question. This quantity is used as the dependent variable in the model formulated by Gulledge and Womer [Ref. 1]. It is this model, which seeks to combine the economist's theories on production rate effects with the learning effects of cumulative production, that the remainder of this paper is devoted to.

III. THE MODEL

The model considered in this thesis was developed by Gulledge and Womer [Ref. 1] in an effort to provide a sensible explanation of the joint effects of learning and production rate on airframe production costs. It was their perception that existing models were extremely limited by an inability to consider production policy changes which often occur prior to or during the course of a production program. Such static models could be of only limited use to a program manager forced to contend with changes in funding and/or production schedules. The thrust of their research was therefore to provide a model capable of capturing the relationship between total program cost and both endogenous and exogenous production rate changes that inevitably occur in the course of a production period. Much of the ensuing description and derivation of the model is extracted from Gulledge and Womer's [Ref. 16] explanation of their revised model.

Womer and Gulledge identified four means by which production scheduling might affect production efficiency and, therefore, cost. The first production cost driver is the usual concept of learning by doing. To help explain this effect and the production cost drivers, Washburn's

[Ref. 4] concept of a production line is incorporated as a frame of reference. Learning by doing affects costs by affecting the efficiency of each position along the production line. That is, as the cumulative number of airframes passing each position increases, experience and efficiency at the position increase and unit costs (or at least labor hours) will decrease. This process implies that at any point in time the experience on the production line could vary significantly.

The second production cost driver is a different learning effect. With the passage of time, learning may occur as a result of events other than experience at a position on the production line. For example, labor hours may be devoted specifically to improving or refining the technique of a particular work station in the early stages of a production program. Later in the program, this may result in increased efficiency independent of experience at a given point on the line. Positions at the end of the line may work more efficiently on the same airframe than positions at the beginning of the line. This effect might also arise from experience at other locations on the production line. In this case, positions late in the production line benefit from the experience of earlier positions and work at these later positions is accomplished more efficiently.

A third production cost driver is the speed of the production line. Unless compensated by learning, an increase in the speed of the line is expected to require more labor at each position on the line. Due to the presence of diminishing returns, such increases in labor are expected to be more than proportional to the increase in speed.

The fourth cost driver is the length of the production line. One means of boosting the delivery rate is to increase the number of positions along the production line. This reduces the workload per position and increases the total amount of work done per unit of time. If alternative length production lines were allowed at the start of production, this driver may not affect unit costs. However, if the length of the line is changed on short notice, unit costs can be adversely affected by crowded facilities and overused tools that hamper production efficiency.

By augmenting a classical homogeneous production function with a learning hypothesis, the model now to be described attempts to capture the above four effects in an estimable and analytic manner. By minimizing the discounted cost of production subject to a production function constraint, the model's output is the optimal time path of resource use. Since relative factor prices are assumed constant over the relevant time period, costs are measured in units of labor, the variable resource. Variables used in the analysis are:

- i = the sequence number of an airframe, $i = 1,2,\ldots,n$
- V = the average number of airframes in process
- t_{si} = the date work begins on airframe i; work on all airframes from the same lot is assumed to start on the lot release date.
- t_{di} = the delivery date for airframe i
- $q_i(t)$ = the production rate at time t on airframe i
- $Q_i(t)$ = the cumulative work performed on airframe i at time t, i.e.,

$$Q_i(t) = \int_{t_{si}}^{t} q(\tau)\,d\tau$$

- $x_i(t)$ = the rate of resource use at time t on airframe i
- δ = a parameter describing learning prior to airframe i
- ε = a parameter describing learning on airframe i
- γ = a parameter describing returns to the variable resources
- α = a parameter associated with decreases in productivity as an airframe nears completion
- ν = a parameter describing returns to the length of the production line
- ρ = the discount rate
- C_i = discounted variable cost of a single airframe

The production function is assumed to be of the following form:

$$q_i(t) = A(i - 1/2)^{\delta}\, Q_i^{\varepsilon}(t)\, (t_{di} - t)^{\alpha}\, x_i^{1/\gamma}(t)\, V^{\nu} \qquad (3.1)$$

where A is a constant. The input x is assumed to be a composite of many inputs whose rate is variable throughout the production period.

This production function is a relatively simple function based upon the four previously mentioned production cost drivers which are combined so as to conform with economic theory and the empirically observed learning phenomenon. When analyzing each term of the function individually, the term $(i - 1/2)^{\delta}$ describes learning by doing in the production of the ith airframe. The terms $Q_i^{\varepsilon}(t)$ and $(t_{di} - t)^{\delta}$ attempt to describe the effects of learning over time. It should be noted that as the delivery date is approached, it is assumed that labor becomes more difficult to substitute for time in the productive process so it is anticipated that both ε and α will be between 0 and 1. The term $x_i^{1/\gamma}(t)$ captures the effect of production rate with γ expected to be greater than 1 and thus consistent with diminishing returns to the variable resource. Finally, the term V^{ν} attempts to capture the effect of the length of the production line. It is assumed that more airframes in the same facility lessens efficiency so that ν should be negative and small.

Assuming a contract written to induce a firm to minimize its discounted costs of production, the problem may be stated as:

$$\text{Minimize } C = \sum_{i=1}^{n} \int_{t_{si}}^{t_{di}} x_i(t) e^{-\rho t} dt \qquad (3.2)$$

subject to:

1) $q_i(t) = A(i - 1/2)^{\delta} Q_i^{\varepsilon}(t)(t_{di} - t)^{\alpha} x_i^{1/\gamma}(t) V^{\nu}$

2) $Q_i(t_{di}) = 1$

3) $Q_i(t_{si}) = 0$

$i = 1, 2, \ldots, n$

Since the total cost of production is monotone nondecreasing and the subproblems are additive, the solution can be obtained by minimizing each of the subproblems. The representative problem for the ith airframe may thus be stated as:

$$\text{Minimize } C_i = \int_{t_{si}}^{t_{di}} x_i(t) e^{-\rho t} dt \qquad (3.3)$$

subject to:

1) $q_i(t) = A(i - 1/2)^{\delta} Q_i^{\varepsilon}(t)(t_{di} - t)^{\alpha} x_i^{1/\gamma}(t) V^{\nu}$

2) $Q_i(t_{di}) = 1$

3) $Q_i(t_{si}) = 0$

The solution procedure is initiated by absorbing the constraints into the objective function. Solving the first constraint for $x_i(t)$ yields the following resource requirement function:

$$x_i(t) = q_i^{\gamma}(t)A^{-\gamma}(i - 1/2)^{-\delta\gamma}Q_i^{-\varepsilon\gamma}(t)(t_{di} - t)^{-\alpha\gamma}V^{-\nu\gamma} \tag{3.4}$$

After substituting into (3.3), the objective function becomes

$$\text{Min } C_i = \int_{t_{si}}^{t_{di}} [q_i^{\gamma}(t)A^{-\gamma}(i-1/2)^{-\delta\gamma}Q_i^{-\varepsilon\gamma}(t)(t_{di}-t)^{-\alpha\gamma}V^{-\nu\gamma}e^{-\rho t}]dt \tag{3.5}$$

To simplify the solution procedure, a transformation that yields one state variable and one control variable is used with the control variable being the time rate of change of the state variable. Let

$$Z(t) = A^{-1}(i - 1/2)^{-\delta}V^{-\nu}Q_i^{1-\varepsilon}(t)/(1 - \varepsilon) \tag{3.6}$$

This implies that

$$z(t) = A^{-1}(i - 1/2)^{-\delta}V^{-\nu}Q_i^{-\varepsilon}(t)q(t) \tag{3.7}$$

Thus, Z(t) is the new state variable and z(t), its time derivative, is the control variable. Substituting into (3.4), we have an expression in terms of the new control variable, i.e.,

$$x_i(t) = z^{\gamma}(t)(t_{di} - t)^{-\alpha\gamma} . \quad (3.8)$$

When substituted into (3.5), this yields the following transformed problem:

$$\text{Minimize } C_i' = \int_{t_{si}}^{t_{di}} [z^{\gamma}(t)(t_{di} - t)^{-\alpha\gamma} e^{-\rho t}]dt = \int_{t_{si}}^{t_{di}} I(z,t)dt \quad (3.9)$$

subject to:

1) $Z(0) = 0$

2) $Z(t_{di}) = A^{-1}(i - 1/2)^{-\delta} V^{-\nu}/(1 - \varepsilon)$

Since the intermediate function, I, does not depend explicity upon the state variable, the Euler equation is

$$\frac{\partial I}{\partial z} = \gamma z^{\gamma-1}(t)(t_{di} - t)^{-\alpha\gamma} e^{-\rho t} = K_o . \quad (3.10)$$

Solving for optimal z(t) yields

$$z(t) = K_1(t_{di} - t)^{\gamma\alpha/(\gamma-1)} e^{\rho t/(\gamma-1)} . \quad (3.11)$$

Substituting this into (3.8) yields a solution to the optimal time path of resource use:

$$x(t) = K_1^{\gamma}(t_{di} - t)^{\alpha\gamma/(\gamma-1)} e^{\gamma\rho t/(\gamma-1)} \quad (3.12)$$

This solution is only of transient interest since the value of constant K_1 is unknown. An optimal expression for x(t) in terms of the variables and parameters of the original problem is needed. To determine the constant of integration, notice that

$$Z(t) = \int_{t_{si}}^{t} K_1(t_{di} - \tau)^{\gamma\alpha/(\gamma-1)} e^{\rho\tau/(\gamma-1)} d\tau + K_2 \tag{3.13}$$

As this is not an easy integral to evaluate, another transformation is useful to simplify the solution. Let $w = \rho(t_{di} - \tau)/(\gamma - 1)$, then $(t_{di} - \tau) = w(\gamma - 1)/\rho$ and $\tau = t_{di} - w(\gamma - 1)/\rho$. The Jacobian of the transformation is $J = d\tau/dw = -(\gamma - 1)/\rho$. Substituting into (3.13), we have

$$Z = \int K_1 [\frac{w(\gamma - 1)}{\rho}]^{\gamma\alpha/(\gamma-1)} e^{[\rho t_{di} - w(\gamma-1)]/(\gamma-1)} J dw \tag{3.14}$$

$$= \int -K_1 \, w^{\gamma\alpha/(\gamma-1)} [\frac{\gamma - 1}{\rho}]^{\{\gamma\alpha/(\gamma-1)\}+1} e^{\rho t_{di}/(\gamma-1)} e^{-w} dw \tag{3.15}$$

Let $K_3 = -K_1 [\frac{(\gamma - 1)}{\rho}]^{\{\gamma\alpha/(\gamma-1)\}+1} e^{\rho t_{di}/(\gamma-1)}$ and we have

$$Z = \int K_3 w^{\gamma\alpha/(\gamma-1)} e^{-w} dw \tag{3.16}$$

Now, the limits of integration must be determined. When $\tau = t$, $w = \rho(t_{di} - t)/(\gamma - 1)$; when $\tau = t_{si}$, and $w = \rho(t_{di} - t_{si})/(\gamma - 1)$. It now follows that

$$Z = \int_{\frac{\rho(t_{di}-t_{si})}{(\gamma-1)}}^{\frac{\rho(t_{di}-t)}{(\gamma-1)}} K_3 w^{\gamma\alpha/(\gamma-1)} e^{-w} dw \tag{3.17}$$

$$= \int_0^{\frac{\rho(t_{di}-t)}{(\gamma-1)}} K_3 w^{\gamma\alpha/(\gamma-1)} e^{-w} dw - \int_0^{\frac{\rho(t_{di}-t_{si})}{(\gamma-1)}} K_3 w^{\gamma\alpha/(\gamma-1)} e^{-w} dw \tag{3.18}$$

These last two expressions can be recognized as incomplete gamma functions; the one on the right is always constant since t does not appear as a variable. We now have

$$Z(u) = -K_3\{\Gamma[\rho(t_{di} - t_{si})/(\gamma - 1), (\gamma\alpha/(\gamma - 1)) + 1]$$

$$- \Gamma[\mu, (\gamma\alpha/(\gamma - 1)) + 1\} + K_4 \tag{3.19}$$

where $\mu = \rho(t_{di} - t)/(\gamma - 1)$. From the constraints for equation (3.9), we know that $Z(0) = 0$. It thus follows that $K_4 = 0$ so it can be removed from (3.19). Also let

$$-K_3 = A^{-1}(i - 1/2)^{-\delta}V^{-\nu}(1 - \varepsilon)^{-1}\Gamma^{-1}[\rho(t_{di} - t_{si})/(\gamma - 1), (\gamma\alpha/(\gamma - 1)) + 1] \qquad (3.20)$$

so that Z also satisfies the final condition:

$$Z(t_{di}) = A^{-1}(i - 1/2)^{-\delta}V(t_{di})^{-\nu}/(1 - \varepsilon)$$

Also note that

$$z(t) = \frac{dZ(u)}{dt} = K_3[\rho\frac{(t_{di}-t)}{(\gamma-1)}]^{\gamma\alpha/(\gamma-1)} e^{-\rho(t_{di}-t)/(\gamma-1)} (-\frac{\rho}{\gamma-1}) \qquad (3.21)$$

After substituting for K_3, the following expression is obtained:

$$z(t) = A^{-1}(i-1/2)^{-\delta}V^{-\nu}(1-\varepsilon)^{-1}\Gamma^{-1}[\rho(t_{di}-t_{si})/(\gamma-1), (\gamma\alpha/(\gamma-1))+1] [\frac{\rho(t_{di} - t)}{(\gamma - 1)}]^{\gamma\alpha/(\gamma-1)} e^{-\rho(t_{di}-t)/(\gamma-1)} (\frac{\rho}{\gamma - 1}) \qquad (3.22)$$

This formulation for optimum z(t) together with (3.11) provides a direct solution for K_1:

$$K_1 = A^{-1}(i-1/2)^{-\delta}V^{-\nu}(1-\varepsilon)^{-1}\Gamma^{-1}[\rho(t_{di}-t_{si})/(\gamma-1), (\gamma\alpha/(\gamma-1))+1] (\frac{\rho}{\gamma-1})^{(\gamma\alpha/(\gamma-1))+1} e^{-\rho t_{di}/(\gamma-1)} \qquad (3.23)$$

Substitution for K_1 in (3.12) yields the following optimum time path of resource use:

$$x_i(t) = B(i - 1/2)^{-\gamma\delta} V^{-\gamma\nu}$$

$$\Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),(\gamma\alpha/(\gamma-1)) + 1]$$

$$(t_{di}-t)^{\alpha\gamma/(\gamma-1)} e^{-\gamma\rho(t_{di}-t)/(\gamma-1)} \tag{3.24}$$

where

$$B = A^{-\gamma}(1-\varepsilon)^{-\gamma}[\rho/(\gamma-1)]^{(\alpha\gamma^2/(\gamma-1))+\gamma} .$$

This is the optimal time path of resource use on any airframe.

Since airframe manufacturing data is generally recorded on a monthly or quarterly basis, the quantity of interest is the total resource used over the appropriate period. The data available for this study are direct labor hours per month so, letting T_1 and T_2 represent the beginning and ending dates for a monthly period, the appropriate expression for airframe i is:

$$X_i(T_2) - X_i(T_1) = \int_{T_1}^{T_2} x_i(t)dt \tag{3.25}$$

Substituting for $x_i(t)$ using equation (3.12), the integral becomes

$$X_i(T_2) - X_i(T_1) = \int_{T_1}^{T_2} K_1^{\gamma}(t_{di}-t)^{\alpha\gamma/(\gamma-1)} e^{\gamma\rho t/(\gamma-1)} dt \tag{3.26}$$

Let $y = \gamma\rho(t_{di}-t)/(\gamma-1)$, then $y(\gamma-1)/\gamma\rho = (t_{di}-t)$ and $t = t_{di} - y(\gamma-1)/\gamma\rho$. Thus, when $t = T_2$, $y = \gamma\rho(t_{di}-T_2)/(\gamma-1)$ and when $t = T_1$, $y = \gamma\rho(t_{di}-T_1)/(\gamma-1)$. It now follows that:

$$X_i(T_2)-X_i(T_1) = \int_{\frac{\gamma\rho(t_{di}-T_1)}{(\gamma-1)}}^{\frac{\gamma\rho(t_{di}-T_2)}{(\gamma-1)}} K_1^{\gamma}\left[\frac{y(\gamma-1)}{\gamma\rho}\right]^{\alpha\gamma/(\gamma-1)} e^{\gamma\rho\left[t_{di}-\frac{y(\gamma-1)}{\gamma\rho}\right]/(\gamma-1)} J dy \tag{3.27}$$

where $J = dt/dy = -(\gamma-1)/\gamma\rho$.

This can be further simplified:

$$X_i(T_2)-X_i(T_1) = \int_{\frac{\gamma\rho(t_{di}-T_1)}{(\gamma-1)}}^{\frac{\gamma\rho(t_{di}-T_2)}{(\gamma-1)}} -K_1^{\gamma}\left(\frac{\gamma-1}{\gamma\rho}\right)^{(\alpha\gamma/(\gamma-1))+1} y^{\alpha\gamma(\gamma-1)} e^{\gamma\rho t_{di}/(\gamma-1)} e^{-y} dy \tag{3.28}$$

$$= -D\int y^{\alpha\gamma/(\gamma-1)} e^{-y} dy \tag{3.29}$$

where $D = K_1^{\gamma}\left(\frac{\gamma-1}{\gamma\rho}\right)^{(\alpha\gamma/(\gamma-1))+1} e^{\gamma\rho t_{di}/(\gamma-1)}$

$$X_i(T_2)-X_i(T_1) = -D\left\{ \int_0^{\frac{\gamma\rho(t_{di}-T_2)}{(\gamma-1)}} y^{[(\alpha\gamma/(\gamma-1))+1]-1} e^{-y}dy - \int_0^{\frac{\gamma\rho(t_{di}-T_1)}{(\gamma-1)}} y^{[(\alpha\gamma/(\gamma-1))+1]-1} e^{-y}dy \right\} \quad (3.30)$$

In this form, the incomplete gamma function is again recognizable and we have

$$X_i(T_2) - X_i(T_1) = -D\left\{ \Gamma\left[\frac{\gamma\rho(t_{di}-T_2)}{(\gamma-1)}, (\alpha\gamma/(\gamma-1)) + 1\right] - \Gamma\left[\frac{\gamma\rho(t_{di}-T_1)}{(\gamma-1)}, (\alpha\gamma/(\gamma-1)) + 1\right]\right\} \quad (3.31)$$

Reversing the order of the last two terms we have

$$X_i(T_2) - X_i(T_1) = D\left\{ \Gamma\left[\frac{\gamma\rho(t_{di}-T_1)}{(\gamma-1)}, (\alpha\gamma/(\gamma-1)) + 1\right] - \Gamma\left[\frac{\gamma\rho(t_{di}-T_2)}{(\gamma-1)}, (\alpha\gamma/(\gamma-1)) + 1\right]\right\} \quad (3.32)$$

Substituting for D and K_1 yields

$$X_i(T_2)-X_i(T_1) = A^{-\gamma}(i-1/2)^{-\gamma\delta}V^{-\gamma\nu}(1-\varepsilon)^{-\gamma}$$

$$\Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),(\gamma\alpha/(\gamma-1))+1]$$

$$(\rho/(\gamma-1))^{\gamma[(\gamma\alpha/(\gamma-1))+1]}e^{-\gamma\rho t_{di}/(\gamma-1)}$$

$$((\gamma-1)/\gamma\rho)^{(\alpha\gamma/(\gamma-1))+1}\, e^{\gamma\rho t_{di}/(\gamma-1)} \qquad (3.33)$$

This can be simplified to

$$Y_i(T_2)-Y_i(T_1) = \beta_0(i-1/2)^{-\gamma\delta}V^{-\gamma\nu}(T_1,T_2)\Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),\beta_1]$$

$$\{\Gamma[\frac{\gamma\rho(t_{di}-T_1)}{\gamma-1},\beta] - \Gamma[\frac{\gamma\rho(t_{di}-T_2)}{\gamma-1},\beta]\} \qquad (3.34)$$

where $\beta_0 = A^{-\gamma}(1-\varepsilon)^{-\gamma}\gamma^{-(\alpha\gamma/(\gamma-1))-1}$
$(\rho/(\gamma-1))^{\gamma[(\gamma\alpha/(\gamma-1))+1]-(\alpha\gamma/(\gamma-1))+1]}$

and $B_1 = (\gamma\alpha/(\gamma-1)) + 1$. This equation quantifies the amount of the variable resource required per month in the production of airframe i. Note that this expression is consistent with the assumptions made concerning the production cost drivers. This resource requirement equation, together with the anticipated range of values for each parameter ($\gamma \geq 1$, $0 \leq \delta \leq 1$, $0 \leq \varepsilon \leq 1$, $0 \leq \alpha \leq 1$, $\nu < 0$), depicts resource usage per month on each airframe to be indirectly related to the airframe sequence number and the dates when work was started and completed on the airframe (representative of the effects of learning) and indirectly related to the number of airframes under construction during the time period (representative of the effects of production rate).

Because of the nature of the data, it is impossible to observe the monthly amounts of resources devoted to any particular airframe. As reported in the data set, what is observable is direct man-hours per job order per month. This means that the observed quantity is

$$\sum_{i=F_J}^{L_J} [X_i(T_2)-X_i(T_1)] = \sum_{i=F_J}^{L_J} \beta_0(i-1/2)^{-\gamma\delta}V^{-\gamma\nu}(T_1,T_2)$$

$$\Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),\beta_1]\{\Gamma[\frac{\gamma\rho(t_{di}-T_1)}{(\gamma-1)},\beta_1] - \Gamma[\frac{\gamma\rho(t_{di}-T_2)}{(\gamma-1)},\beta_1]\} \tag{3.35}$$

where F_J and L_J are the sequence numbers of the first and last airframes in Job Order J.

This functional form represents the aggregate of resource usage per month in the production of all airframes in Job Order J and facilitates the generation of a time path of resource use. Using monthly labor hours and airframe delivery dates as the variables in the equation, nonlinear regression using Marquardt's compromise [Ref. 17] is used to generate estimates for the parameters of equation (3.35). When equation (3.35) has been fully parameterized with these estimates, substitution of job order release and delivery dates and airframe sequence numbers allows prediction of monthly labor hour requirements. The following chapters are devoted to reporting the empirical results of the model and assessing how well it explains the data.

IV. THE DATA

Based on both its availability and suitability for evaluating the model's potential as a tool for a program manager, cost data for the F-4 Phantom II was used in this analysis. The F-4 was designed and built by the McDonnell Aircraft Company of St. Louis, Missouri in response to the Navy's requirements for a carrier-based aircraft capable of fleet air defense. First flown in May, 1958, the F-4 became the first Mach 2-plus carrier-based fighter by incorporating numerous advanced designs and technologies. So impressive was its performance that later versions of the F-4 were purchased by both the U.S. Air Force and Marine Corps as well as over ten foreign air forces. In total, thirteen versions of the F-4 were produced with more than 4000 aircraft delivered between 1958 and 1979.

Of immediate concern with such an extensive and heterogeneous data base are the interrelationships between different versions of the aircraft. In particular, the extent of learning and the effects on the production rate parameter are somewhat obscured by having multiple versions of the aircraft under simultaneous production. While not discounting this potential problem, the necessity for resolving it is relieved by evaluating the model's usefulness from the perspective of a Navy program manager who quite possibly could encounter this same problem in his use of the model. One possible scenario would be where a program manager has cost data on an early version of an airframe purchased by the Navy and must develop a funding profile for buying an improved version based on the lot release and delivery dates specified in the manufacturer's production contract. As it would not be unusual for the early version or the improved version or both to be produced concurrently with other contracts, it would be useful to know the model's usefulness in this situation.

The F-4 program provides a particularly well suited data base for accomplishing the proposed objective. The Navy purchased two versions of the F-4, the F-4B and the F-4J. In all, 660 F-4B airframes were accepted between February, 1958 and January, 1967 and 552 F-4J airframes were accepted between October, 1964 and December, 1971. The F-4B cost data will be used as the data base for estimating the model's parameters. These data were drawn from two sources. Direct man hours per month for each job order of F-4 production are reported in the McDonnell Aircraft Company's document "Report 7290, F-4 Cost Data" [Ref. 18]. Aircraft acceptance dates were obtained from the OASD publication "Acceptance Rates and Tooling Capacity for Selected Military Aircraft" [Ref. 19] and used in lieu of delivery dates which were not available. Since the exact acceptance dates were unavailable, for purposes of research, airframe acceptances were assumed to be distributed uniformly over the month of acceptance.

Several problems were encountered in compiling the data to use with the model. The first job order of the F-4B production run accumulated in excess of 172,000 labor hours before its reported release date. These labor hours were reported as a lump-sum which cannot be satisfactorily explained. As a result, these labor hours were not used in the model; the data which were used begin with the month that this first job order was released for production.

Each job order also had a small number of manhours expended in months after all the airframes within that job order had been accepted. These hours were aggregated and then distributed evenly throughout the preceding months of the job order.

With these adjustments, 233 observations of labor hours recorded over 108 months in the production of 660 airframes remained. These observations, together with the average number of airframes under production per month, form the data for study as reported in the appendix.

V. EMPIRICAL RESULTS

As formulated in Chapter III, the proposed model is that manhours per job order per month are equal to

$$\sum_{i=F_J}^{L_J} [X_i(T_2)-X_i(T_1)] = \sum_{i=F_J}^{L_J} \beta_0 (i-1/2)^{-\gamma\delta} V^{-\gamma\nu}(T_1,T_2)$$

$$\Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),\beta_1]\{\Gamma[\frac{\gamma\rho(t_{di}-T_1)}{(\gamma-1)},\beta_1] - \Gamma[\frac{\gamma\rho(t_{di}-T_2)}{(\gamma-1)},\beta_1]\}$$

The regression estimates of the model's parameters are presented in Table I.

TABLE I

PARAMETER ESTIMATES

Parameter	Estimate	Standard Error
β_o	2.509	0.665
β_1	3.031	0.325
δ	0.462	0.024
γ	1.006	0.001
ν	-0.200	0.067

All parameter estimates are significantly different from zero and consistent with apriori assumptions. As one of the model's key assumptions is the existence of an inverse relationship between production rate and unit costs, attention is first directed to the estimated value for γ. The estimated value of 1.006 indicates the existence of decreasing returns to labor inputs. That is, increases in the rate of production will increase labor requirements (and hence unit costs). The estimated value of δ corresponds to a 73 per cent learning curve. While this percentage may be slightly low by industry standards, it is not all that surprising due to the concurrent production of other models of the F-4. Some learning in the production of these other models undoubtedly benefited the F-4B program and contributed to the observed 73 per cent learning curve. The small negative value for ν confirms expectations about the effects of facility size and crowding. β_o is a scaling parameter and appears consistent with earlier studies. The estimated values of β_1 implies α and ε values of 0.01 which, while small, are within the range of apriori assumptions.

Graphical analysis demonstrates the time path of resource use generated by the model to be a reasonable one. Figure 2 illustrates the time path of labor use predicted by the model and the actual labor hours used. Figure 3 compares a 90 per cent confidence interval for the predicted

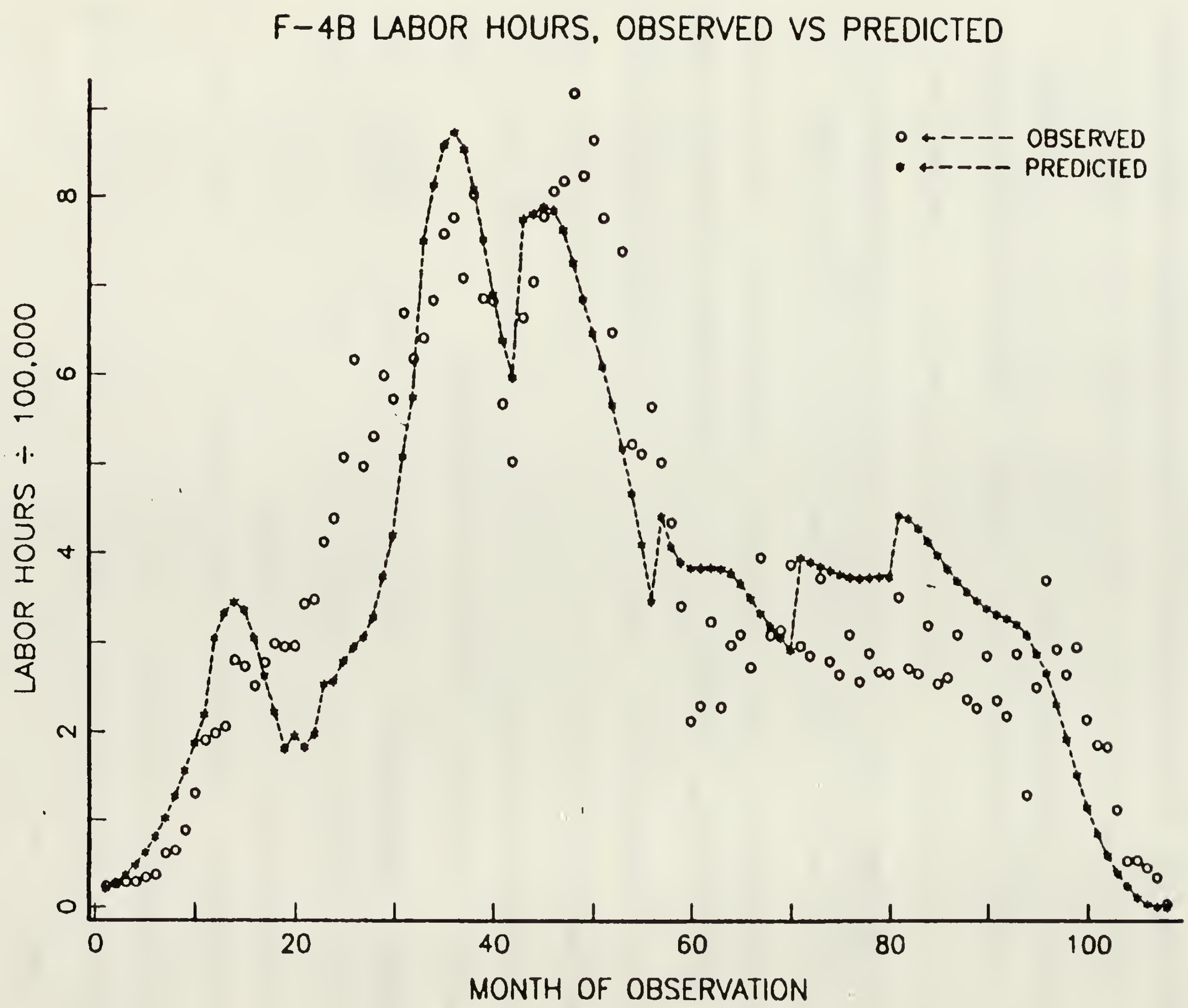


Figure 2. F-4B Labor Hours, Observed vs Predicted

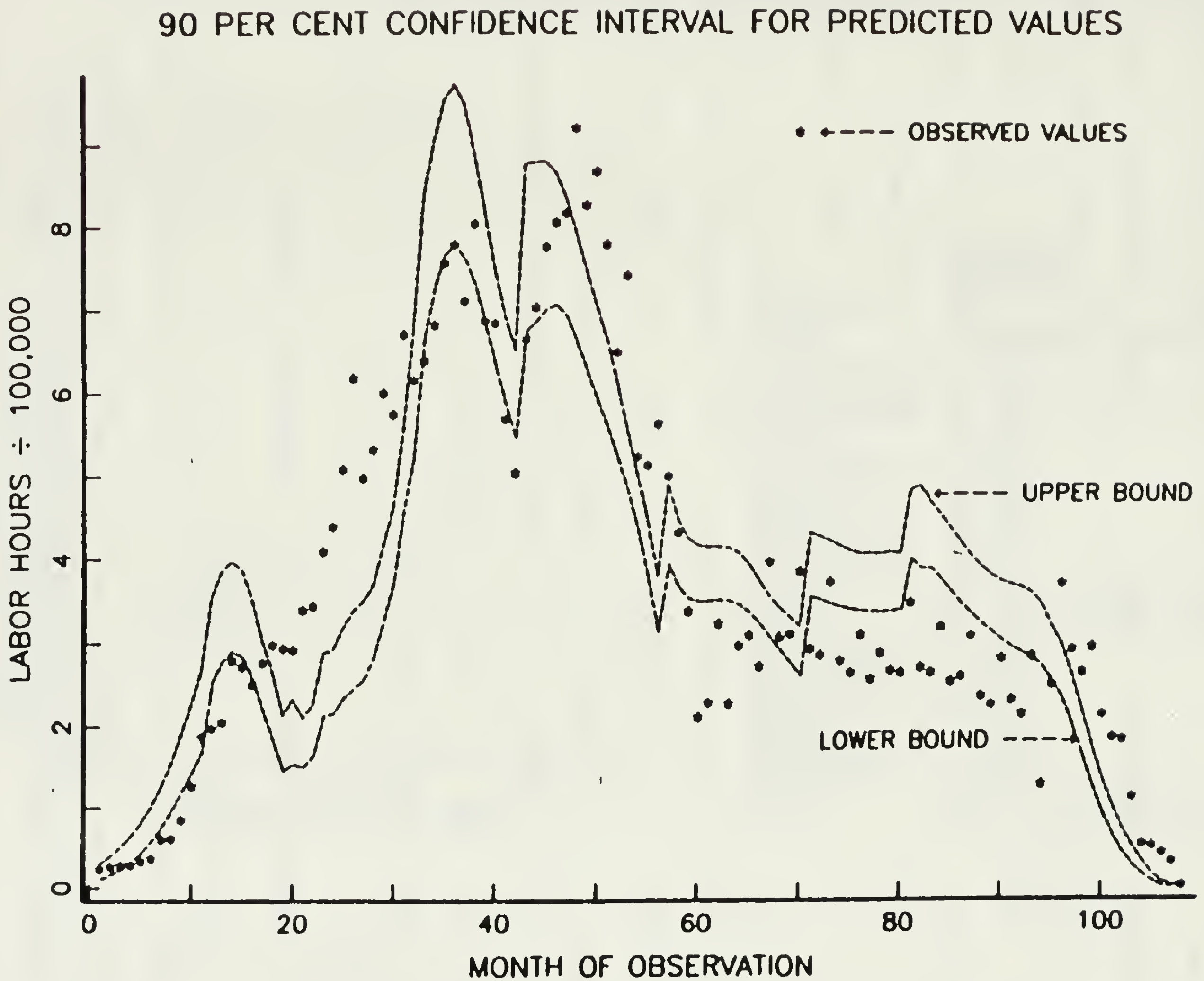
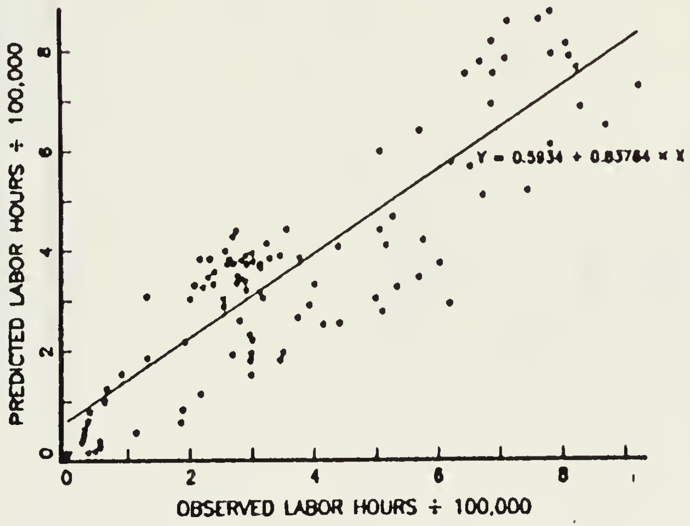


Figure 3. 90 per cent Confidence Interval for Predicted Values

labor hours with the observed labor hours. The model does fit the data quite well as evidenced by the R^2 value of .76 obtained by squaring the XY correlation coefficient reported in Figure 4. This figure shows that plotting the observed values against the predicted values yields approximately a 45 degree line which further illustrates the model's success in explaining the variability of the data. Relatively large and unexplained deviations between the observed and predicted values are apparent during roughly the first and last thirds of the production run (approximately months 19-31 and 69-96). Disassembling Figure 2 into individual job orders (Figures 5-13) reveals that most of the deviation noted during the first third of the production run can be attributed to Job Order 692. This job order is characterized by the predicted time path lagging several months behind the observed. A factor of unknown significance is that this was the first sizable job order to be released. The second area of deviation corresponds with a very large release of another model of the F-4 which may have had unpredictable effects on the regression output.

Analysis of the residuals shows them to be essentially normal with no extreme outliers noticeable. Residual analysis was conducted on two levels--the 233 observations available before aggregating the nine job orders and the 108 observations formed by combining the job orders into the

SCATTER PLOT OF OBSERVED VS PREDICTED LABOR HOURS



SCATTER PLOT TABLE	
X	:OBSERVED
Y	:PREDICTED
SELECTION	:ALL
X LABEL	:OBSERVED LABOR HOURS ÷ 100,000
Y LABEL	:PREDICTED LABOR HOURS ÷ 100,000
NO. OF ELEMENTS	:100
CORRELATION XY	:0.87428 $R^2 = .76$
RK CORRELATION	:0.7608 T=12.417
X MEAN	:3.6772
STD. DEVIATION	:2.2998
5-PERCENTILE	:0.33922
25-PERCENTILE	:2.2852
MEDIAN	:2.9614
75-PERCENTILE	:5.237
95-PERCENTILE	:8.0337
X MIN.	:0.04414 0.23904 0.26792
X MAX.	:9.2011 8.6654 8.2533
Y MEAN	:3.6735
STD. DEVIATION	:2.2034
5-PERCENTILE	:0.24112
25-PERCENTILE	:2.218
MEDIAN	:3.5857
75-PERCENTILE	:4.2941
95-PERCENTILE	:7.0987
Y MIN	:0.00040071 0.008576 0.040087
Y MAX	:8.7471 8.5989 8.5527

Figure 4. Scatter Plot of Observed vs Predicted Labor Hours

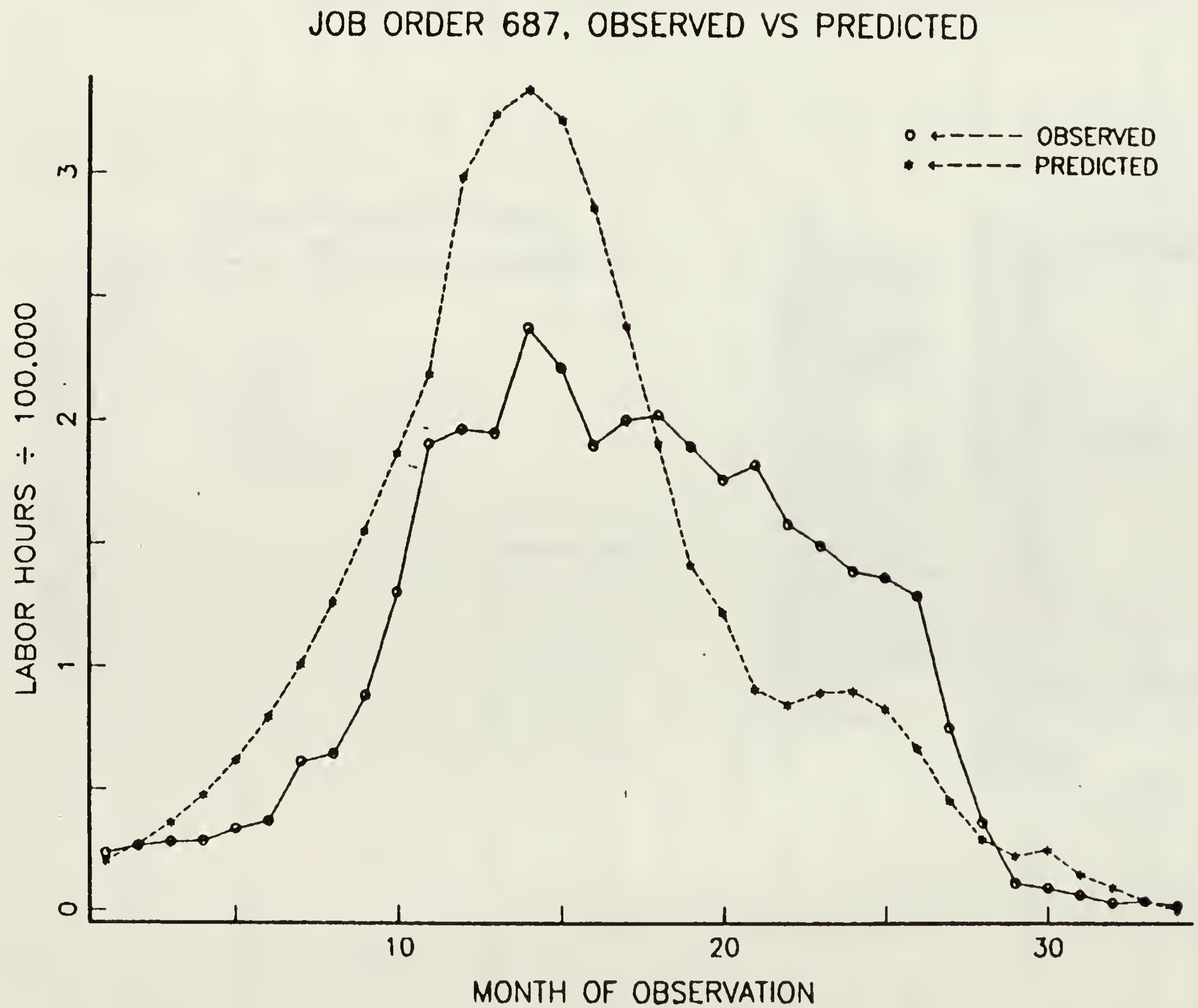


Figure 5. Job Order 687, Observed vs Predicted

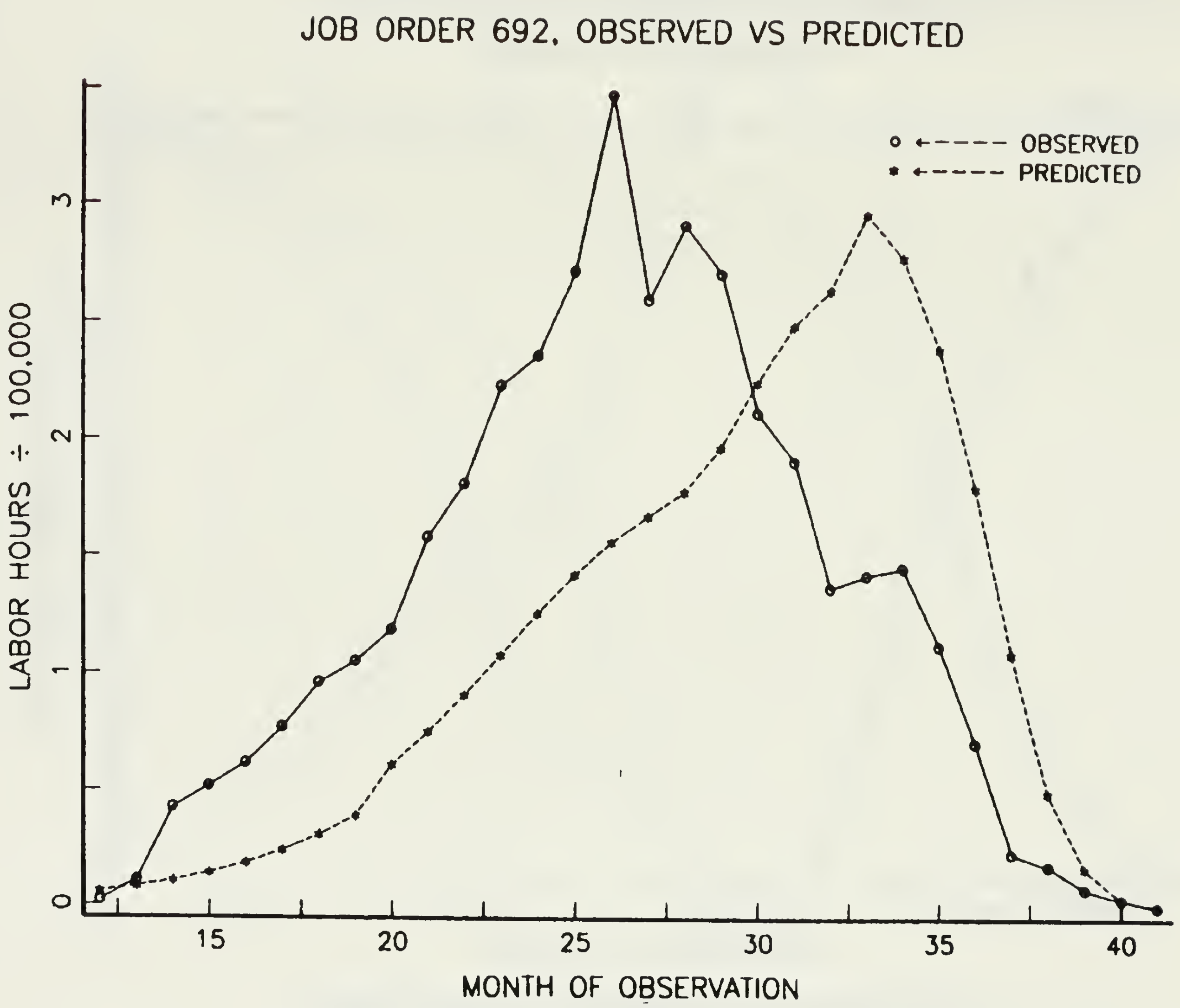


Figure 6. Job Order 692, Observed vs Predicted

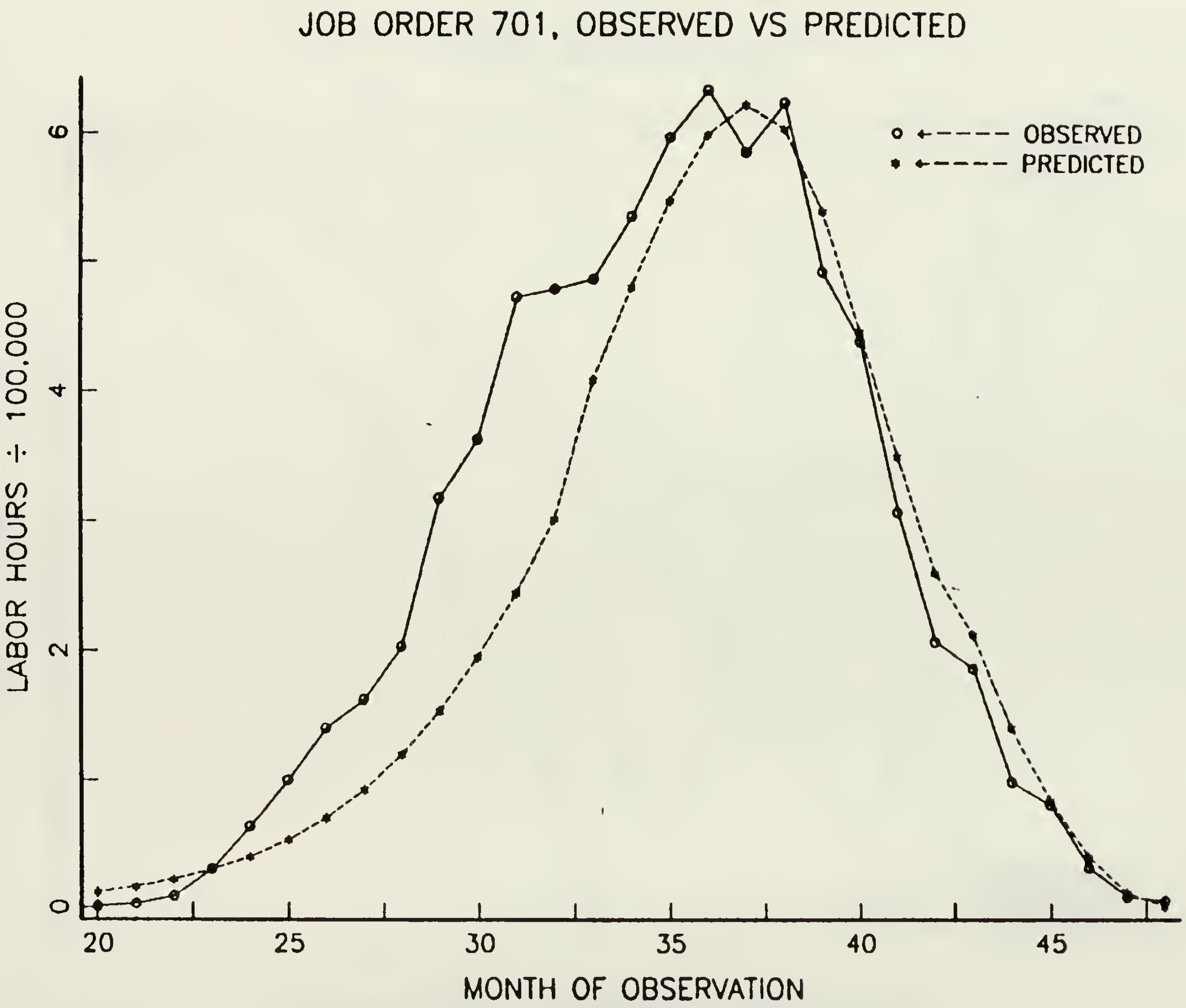


Figure 7. Job Order 701, Observed vs Predicted

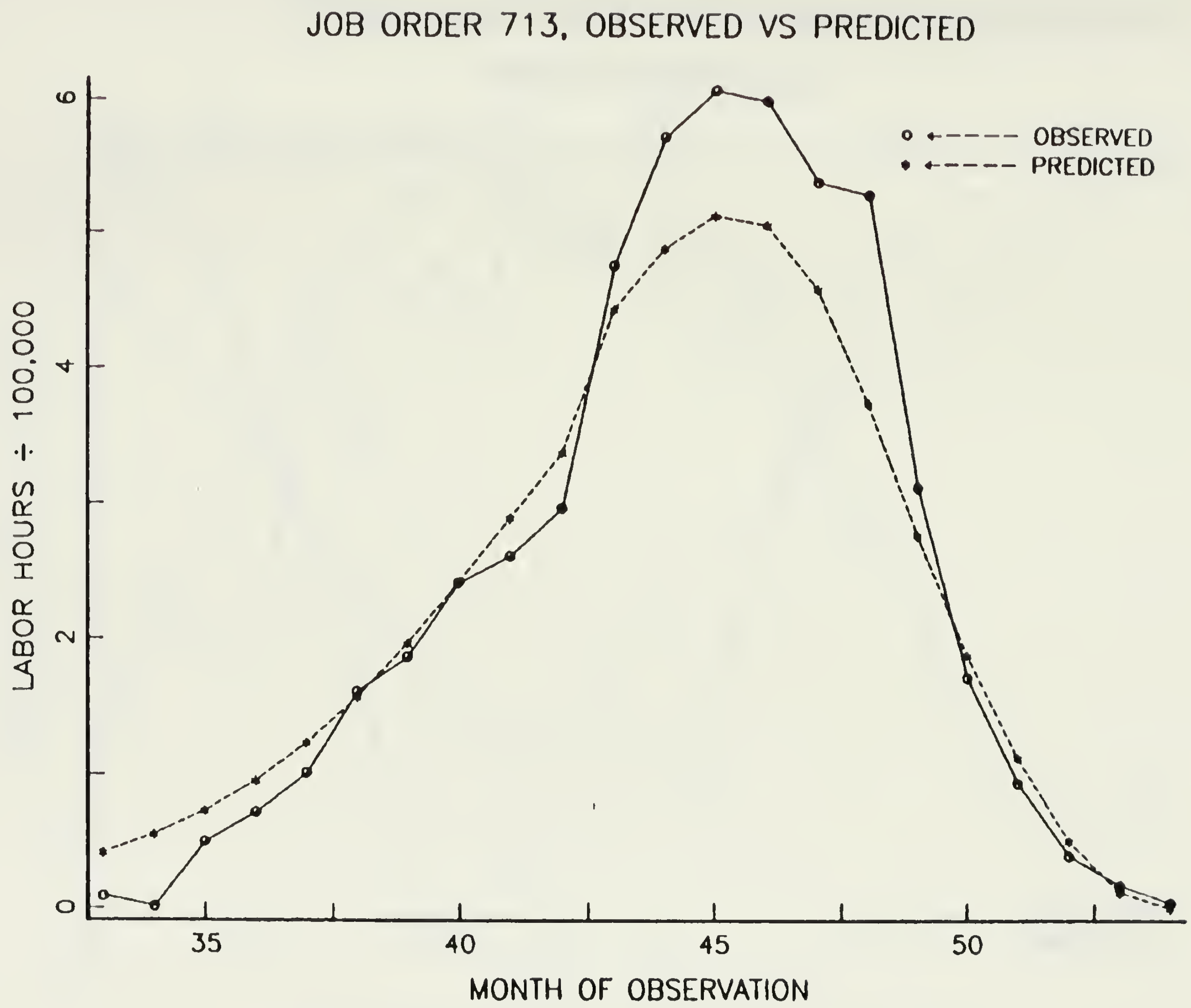


Figure 8. Job Order 713, Observed vs Predicted

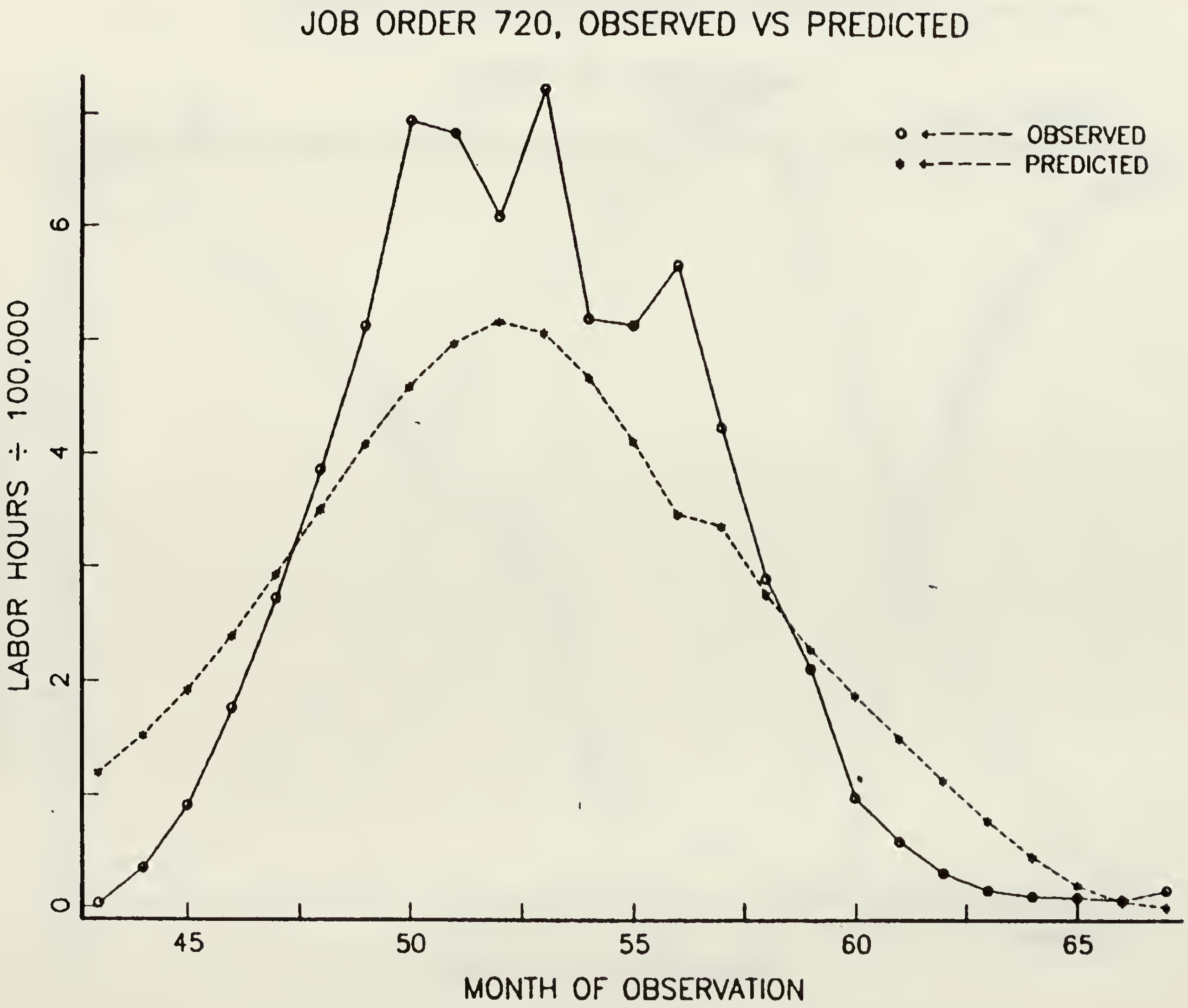


Figure 9. Job Order 720, Observed vs Predicted

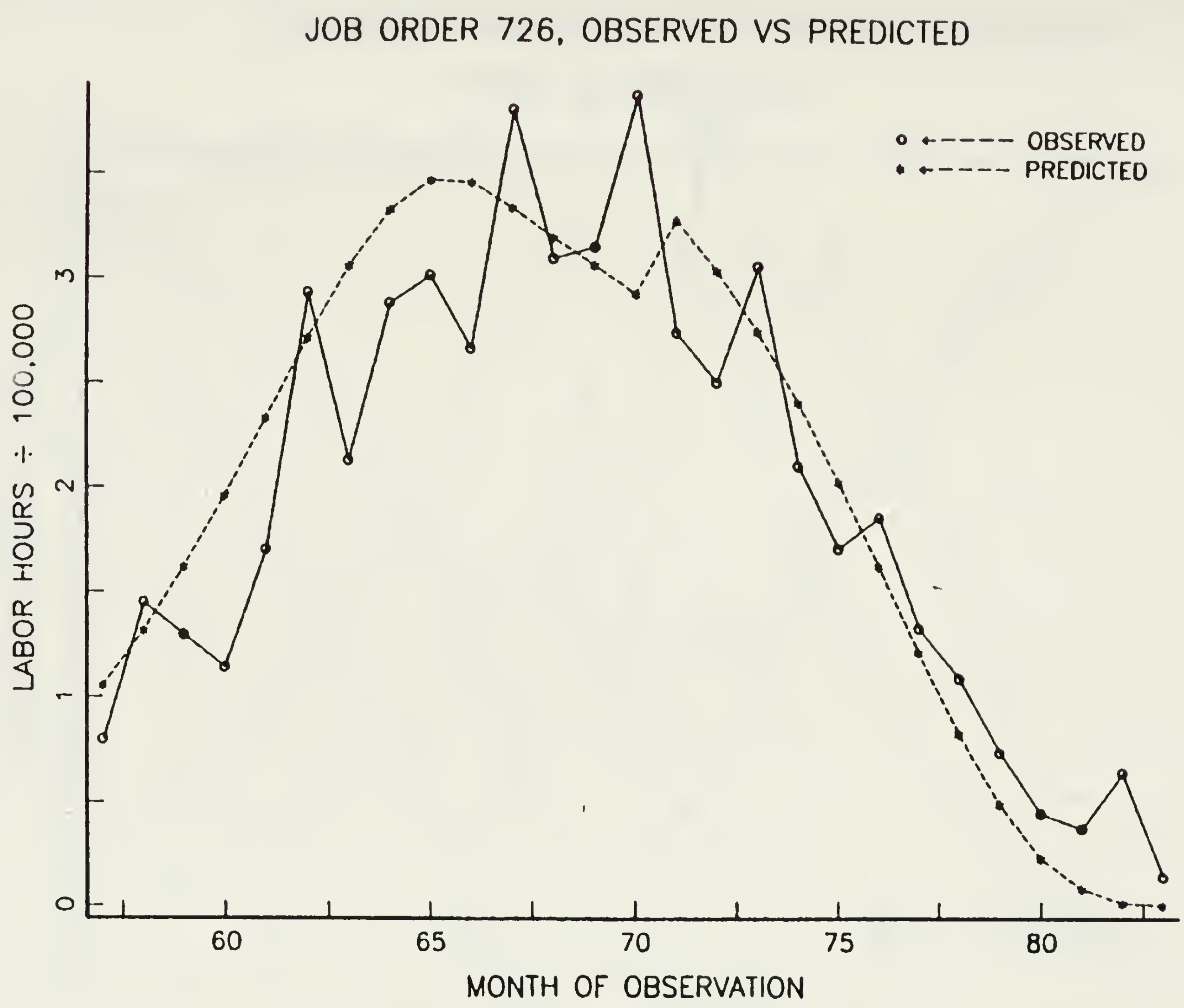


Figure 10. Job Order 726, Observed vs Predicted

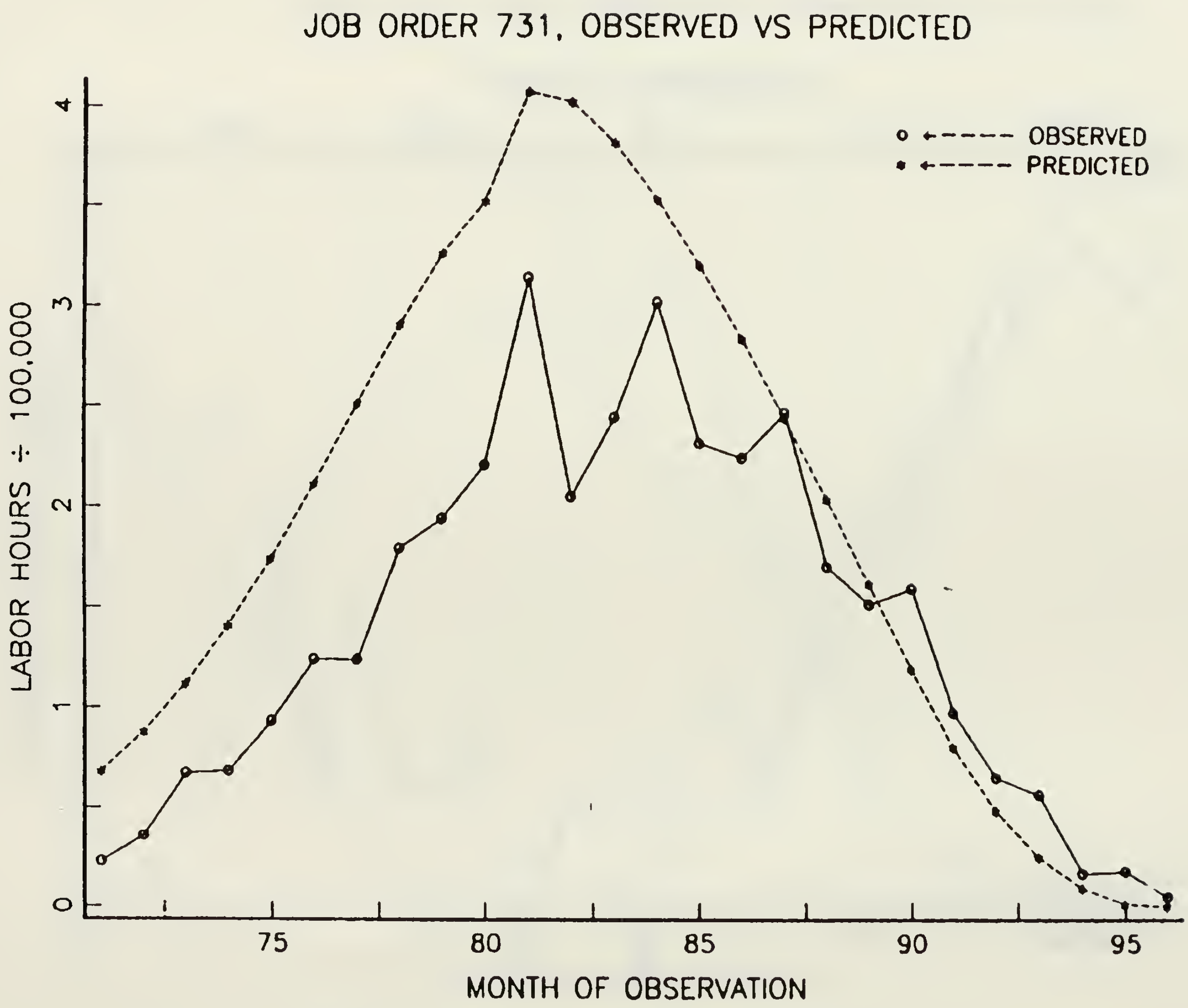


Figure 11. Job Order 731, Observed vs Predicted

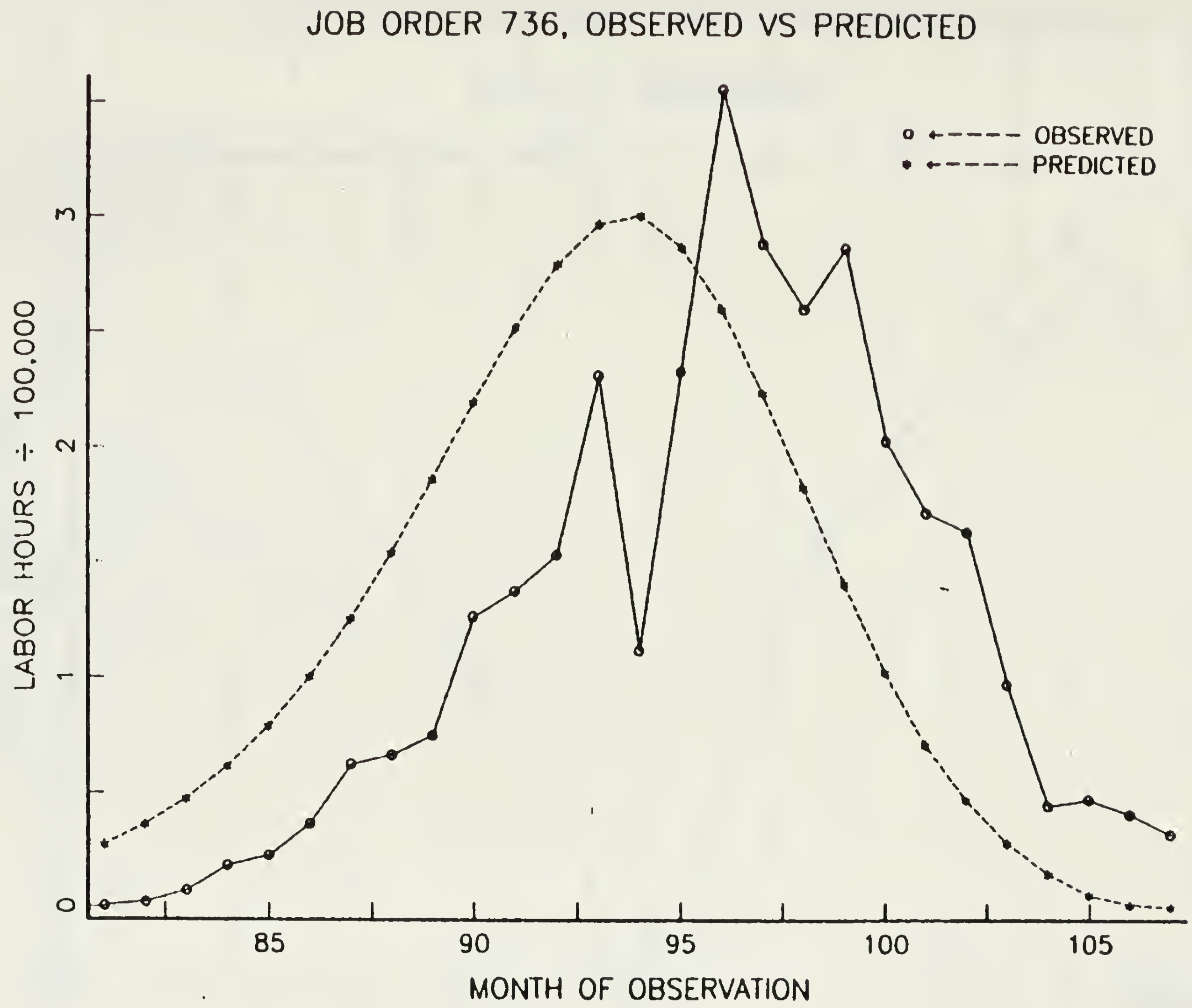


Figure 12. Job Order 736, Observed vs Predicted

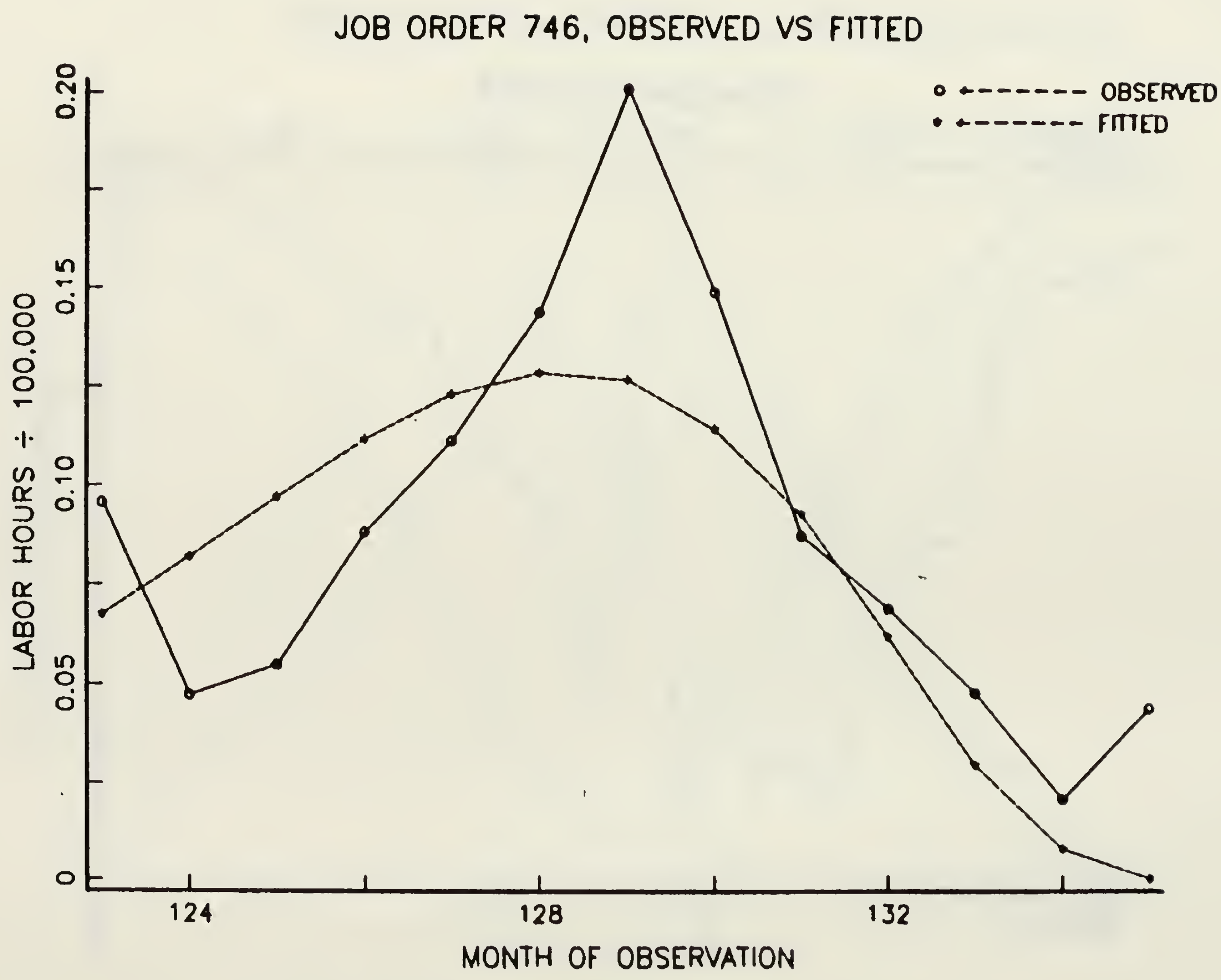


Figure 13. Job Order 746, Observed vs Predicted

time path shown in Figure 2. Figures 14-18 depict the larger set of residuals. These residuals generally appear random and symmetric with a sample mean of 0.002 and some positive skewness evident. When plotted against the dependent variable, the sign and magnitude of the residuals do exhibit a slight positive relationship with the observed amount of labor hours. The statistical table of Figure 18 contains several goodness of fit statistics which generally do not contradict, at a .05 significance level, a hypothesis of normally distributed residuals. To test for heteroskedasticity, the residuals were divided in half and a hypothesis of equal variances was tested using an F test (see Mood, Graybill, and Boes [Ref. 20]). An F statistic of 1.19 (associated significance level = 0.34) was calculated. This supports the conclusion that the variance of the residuals is constant. This conclusion is further supported by the lack of any noticeable pattern upon visual inspection of the residuals in Figures 14-17.

Similar analysis of the smaller set of 108 residuals shows that they also are fairly normal with a sample mean of 0.004. These residuals are not as symmetric as the larger set as evidenced by the higher measure of skewness reported in the statistical table of Figure 21. While these residuals are perhaps not as nearly normal in appearance as the larger set, a hypothesis of normally distributed

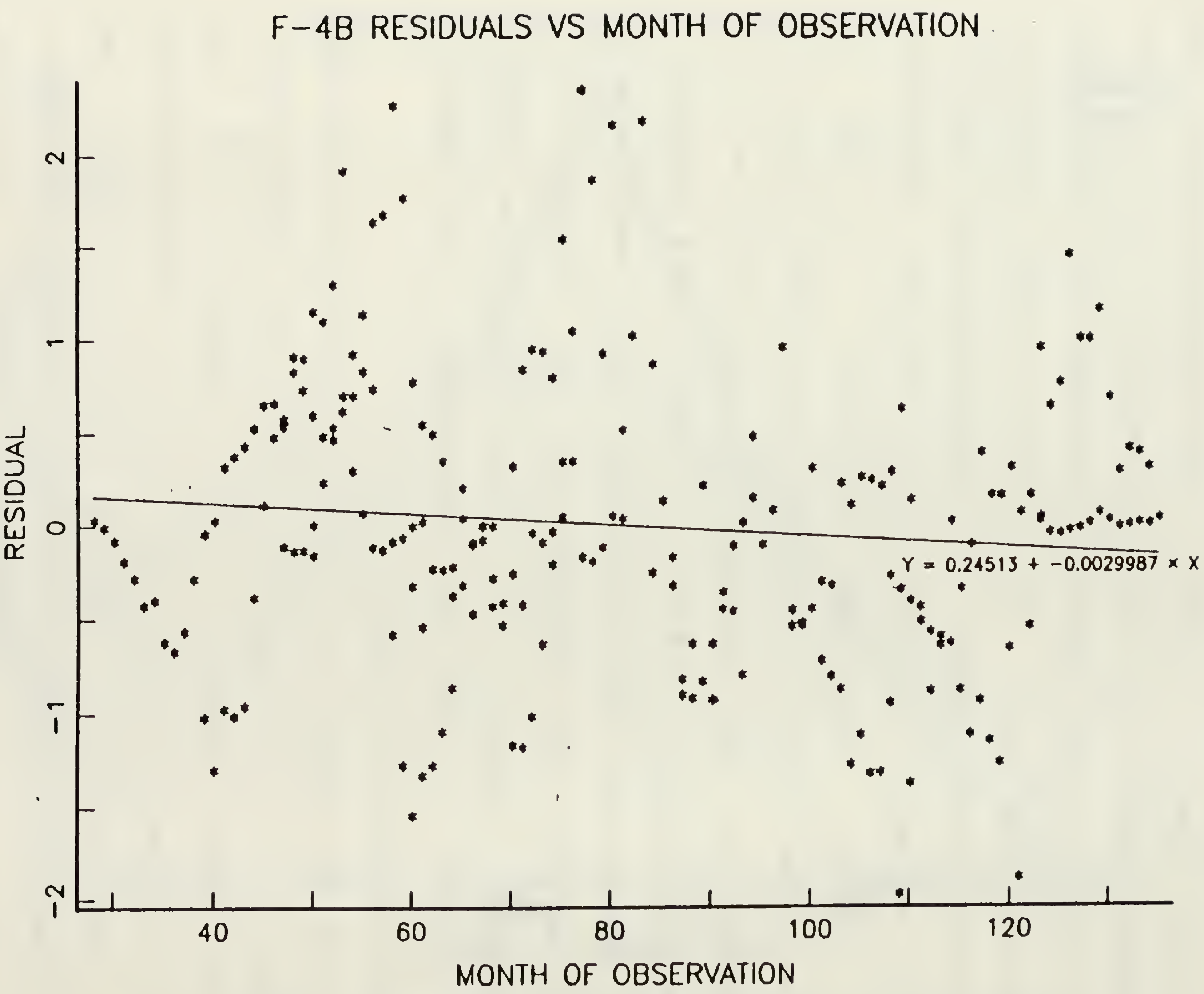


Figure 14. F-4B Residuals vs Month of Observation

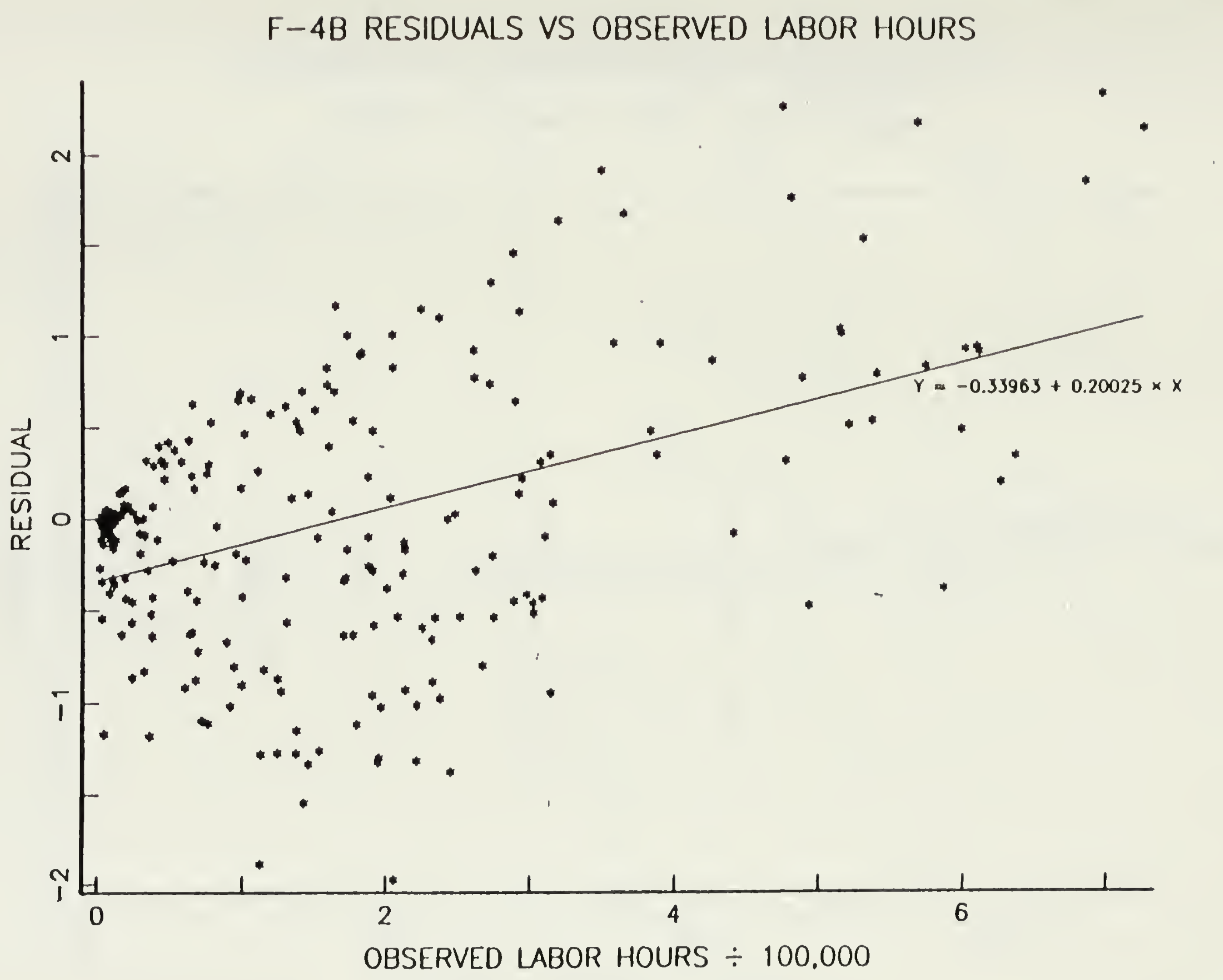


Figure 15. F-4B Residuals vs Observed Labor Hours

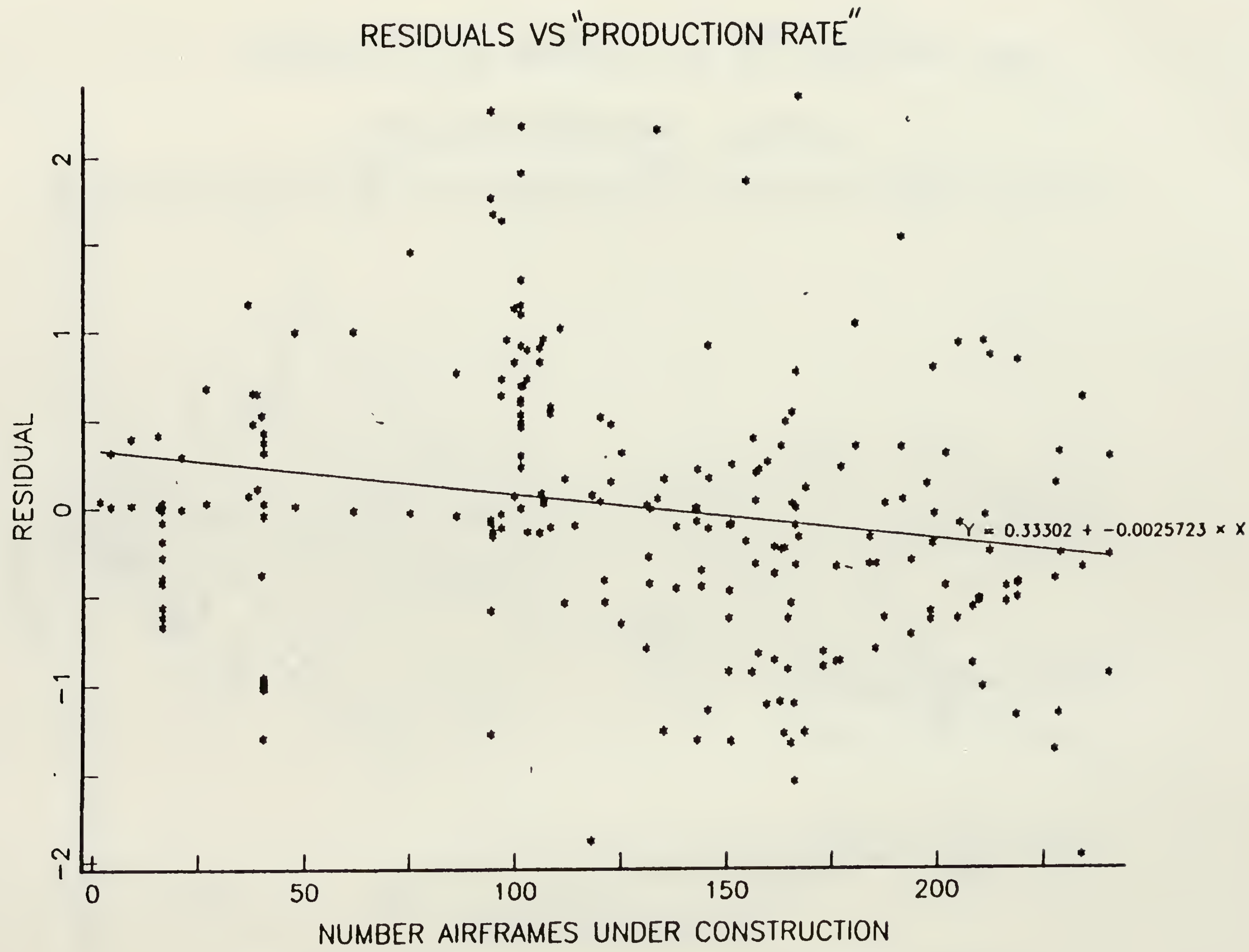


Figure 16. Residuals vs "Production Rate"

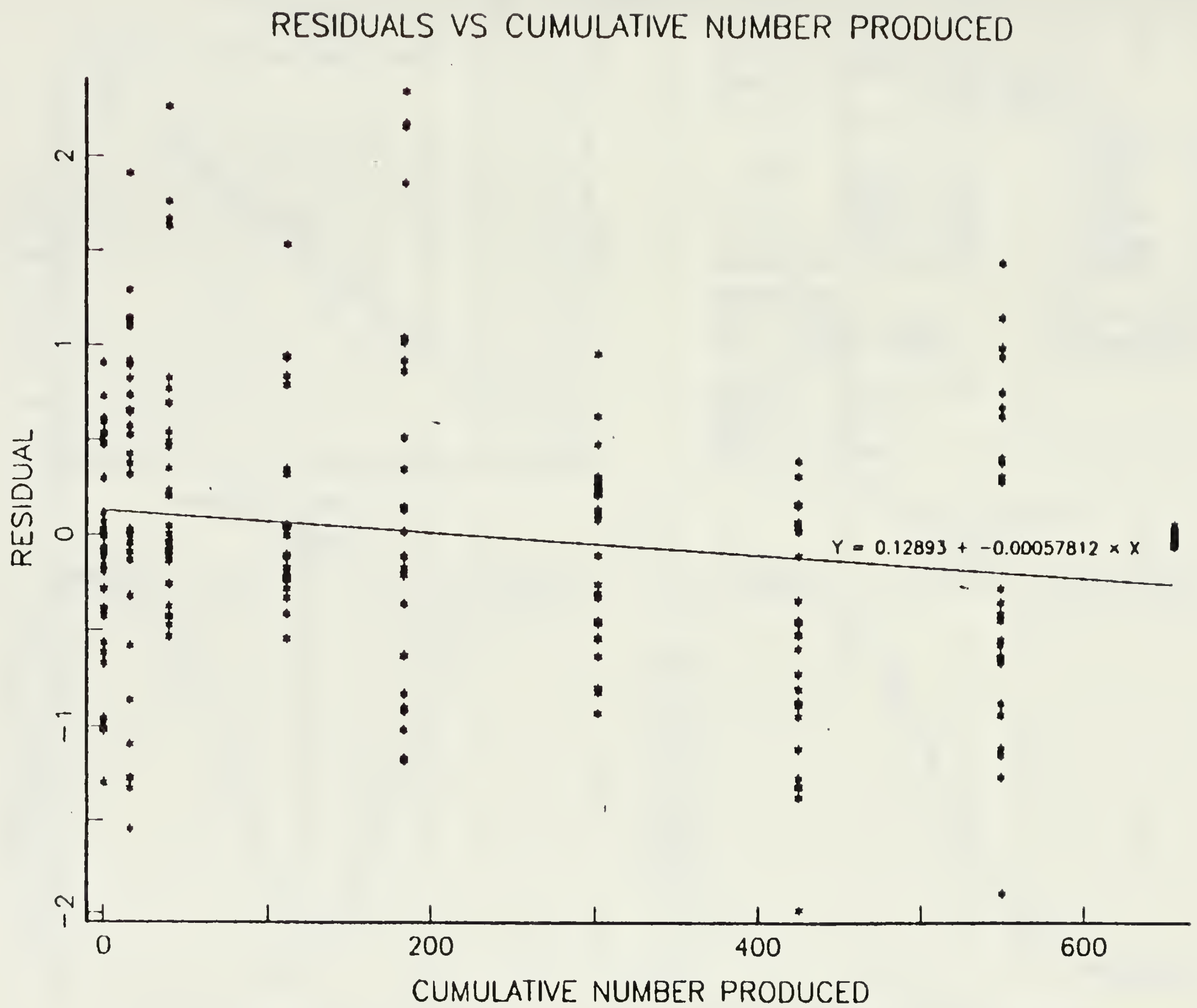


Figure 17. Residuals vs Cumulative Number Produced

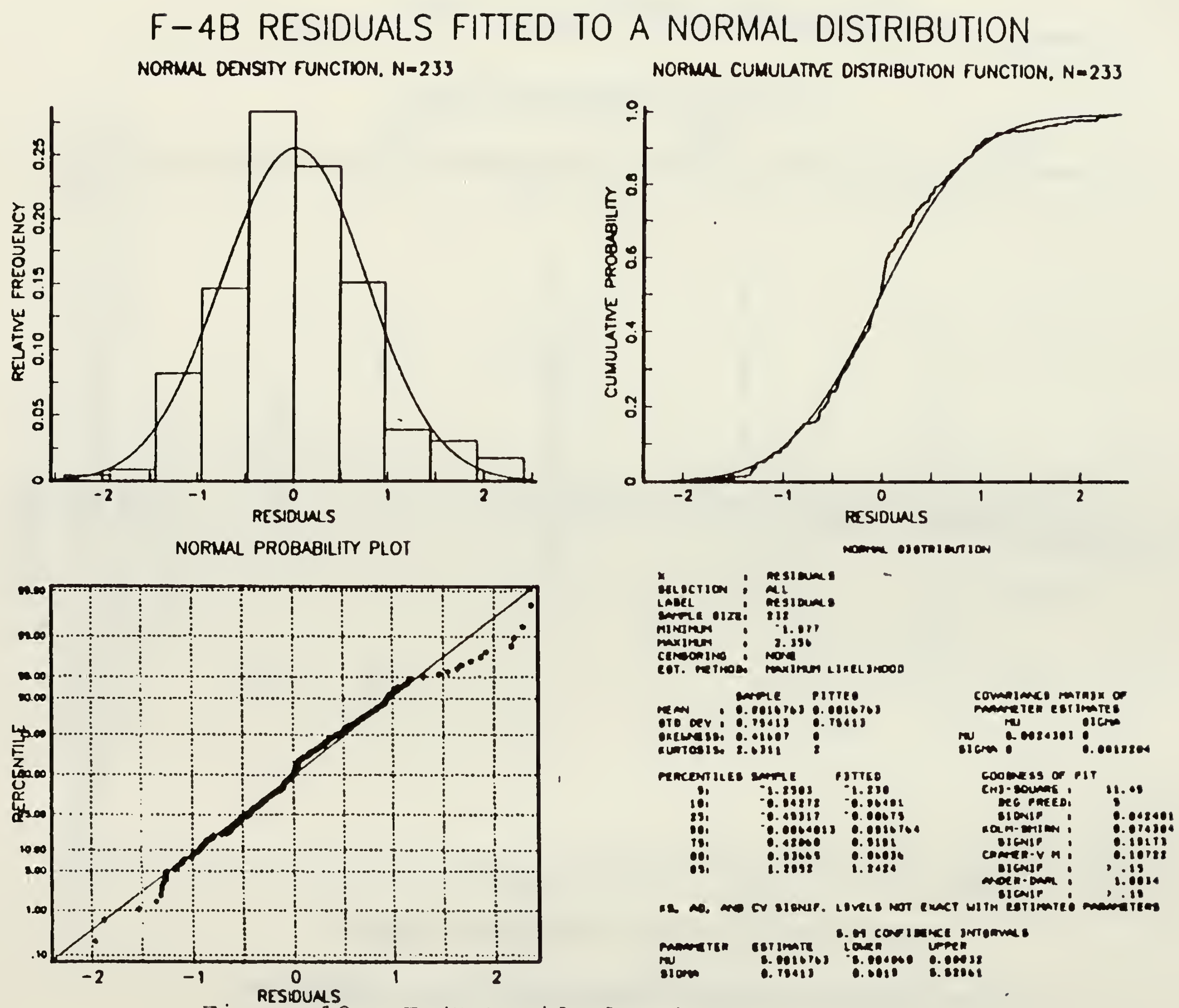


Figure 18. F-4B Residuals Fitted to a Normal Distribution

residuals is again generally supported at the .05 significance level by the goodness of fit statistics. In this case, the test for heteroskedasticity yielded an F-statistic of 1.55 (associated significance level = 0.11) which again supports a conclusion of constant variance. In this case, however, there appears to be a cyclical pattern in the residuals that is most apparent in Figure 19.

The cyclical pattern noted above suggests that a test for autocorrelation is in order. In contrast to the larger set, a test for autocorrelation in the aggregated residuals is meaningful since they are derived from an entirely sequential time series. The Durbin-Watson test (see Judge et. al. [Ref. 21]) was used to test for first order autocorrelation in the aggregated set of 108 residuals as well as in each of the individual job orders. Considering the four production cost drivers as the independent variables and including a constant term (i.e., K = 5), a null hypothesis of no autocorrelation was tested against an alternative hypothesis of positive autocorrelation. As shown in Table II, the Durbin-Watson test statistic was, with but two exceptions, less than the lower critical value, d_L, and thus indicative of a positive autocorrelation problem. Since a nonlinear model was used, transformations of the variables to eliminate this problem are inappropriate and the autocorrelation must therefore be included in this model.

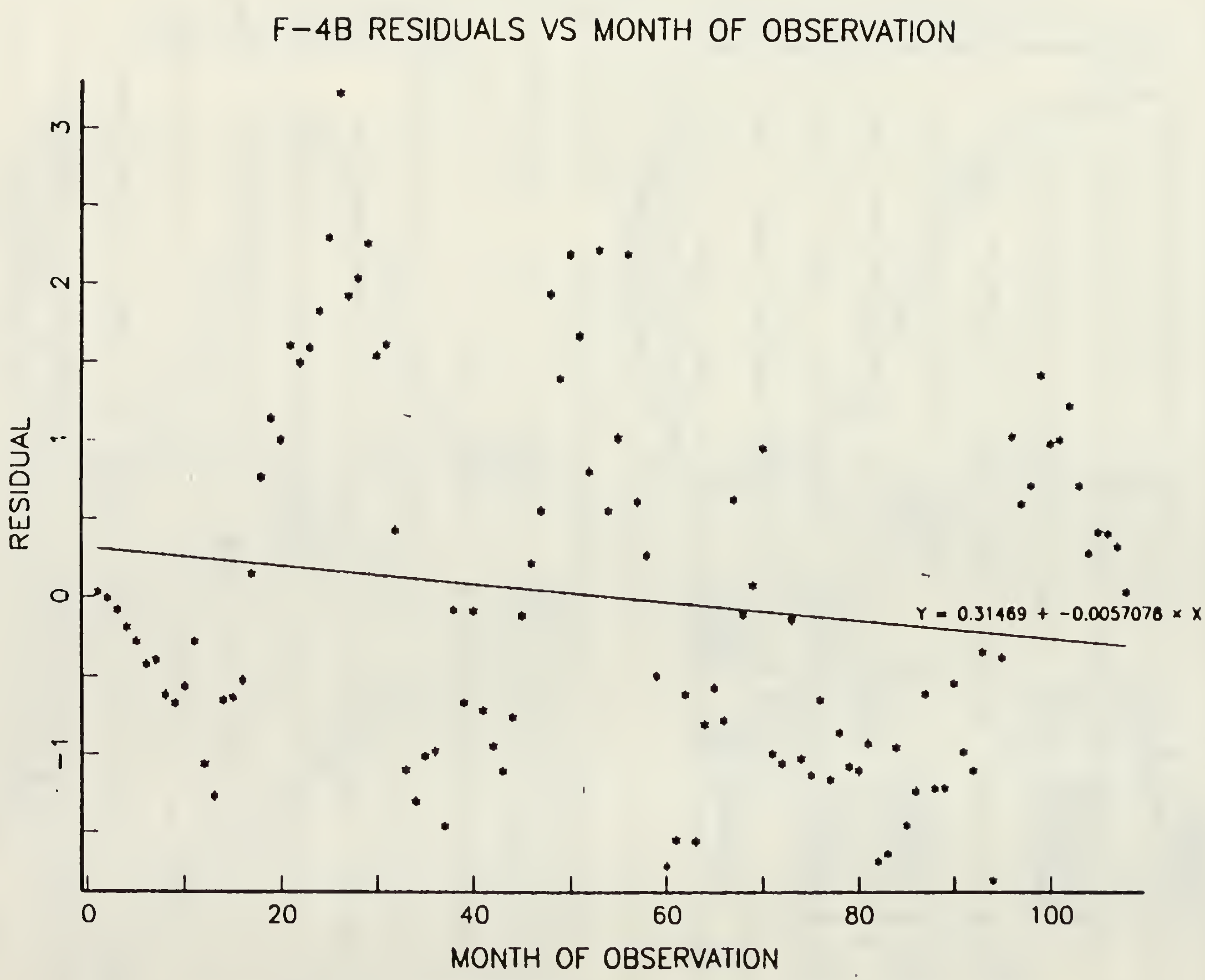


Figure 19. F-4B Residuals vs Month of Observation

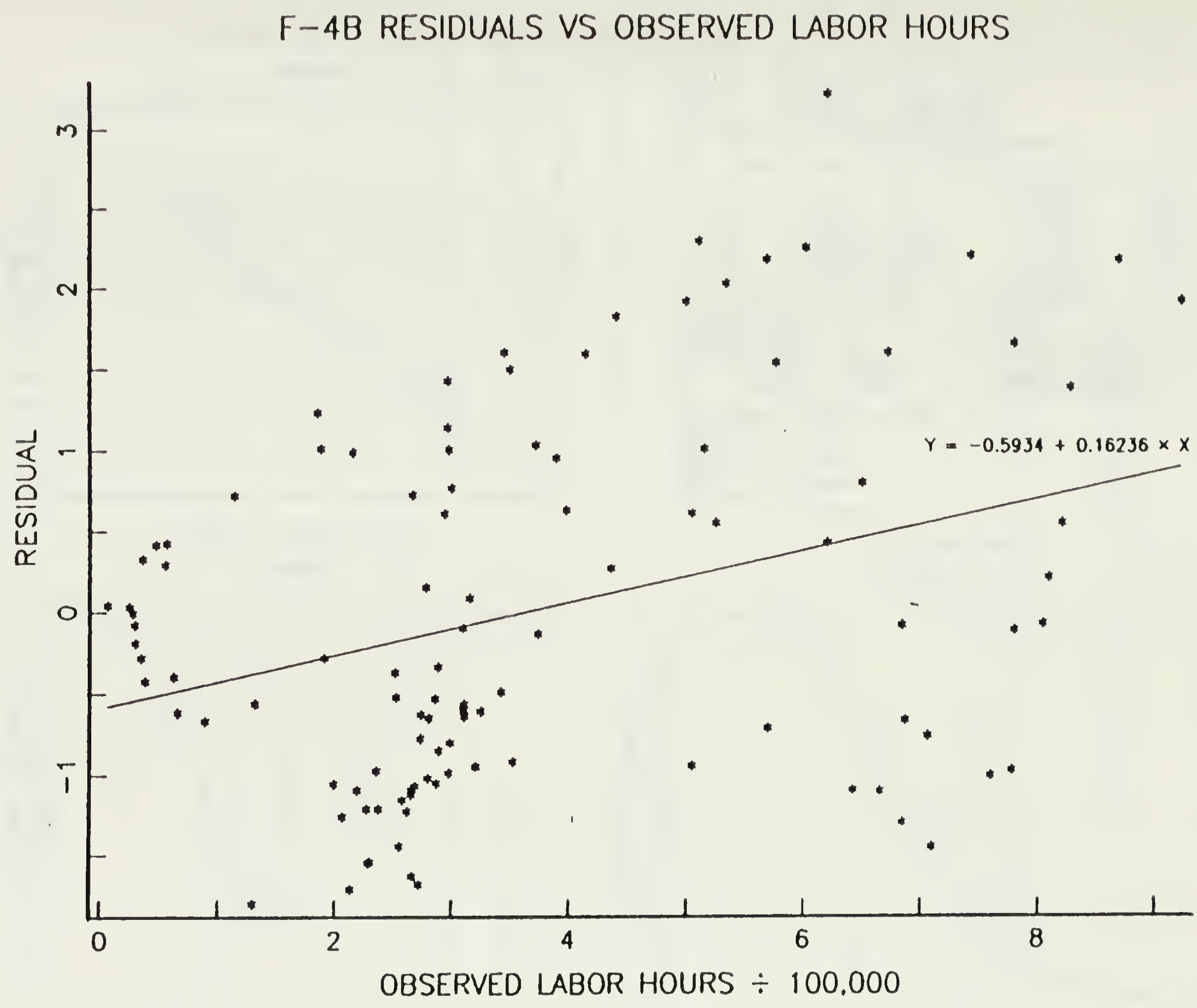


Figure 20. F-4B Residuals vs Observed Labor Hours

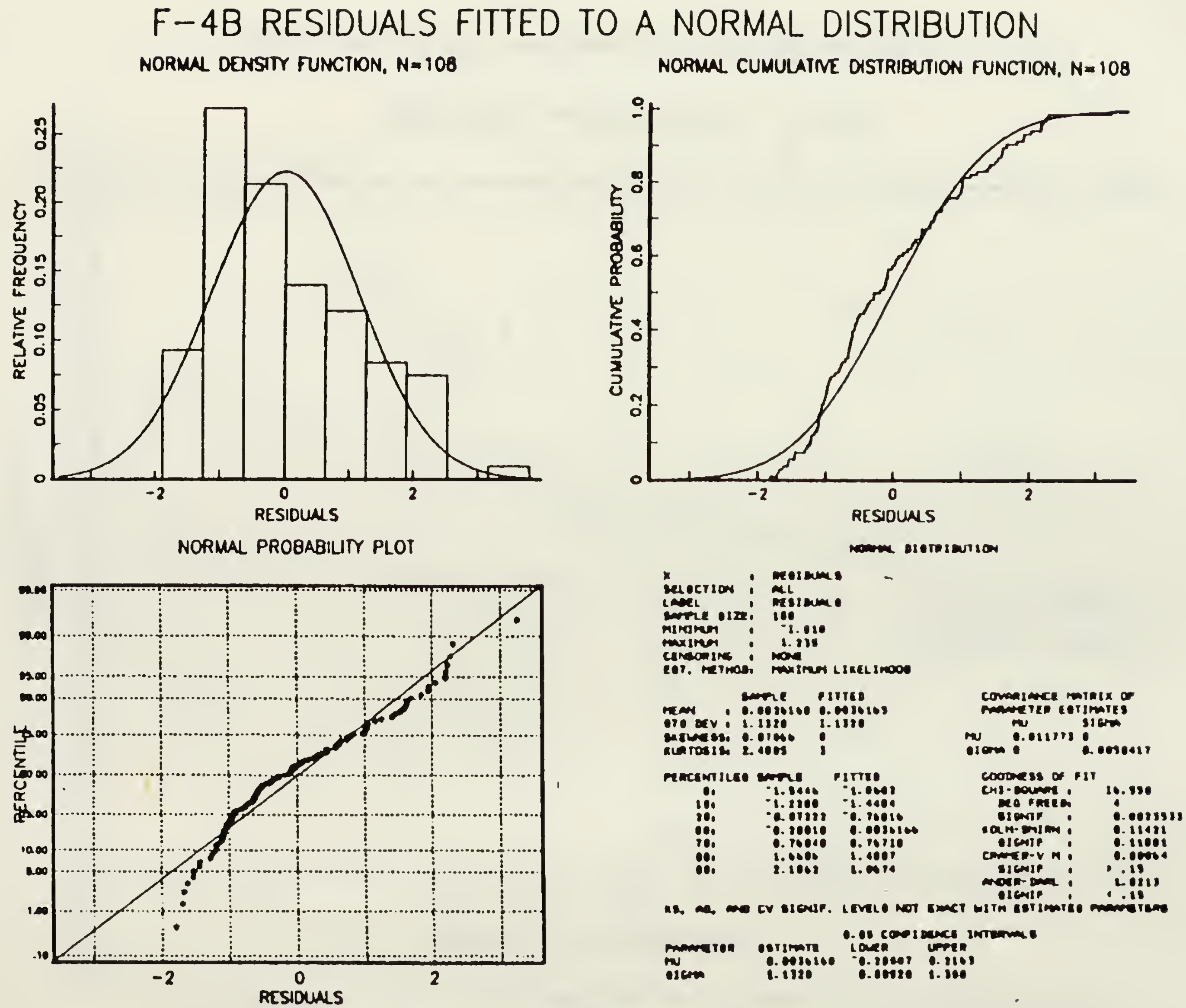


Figure 21. F-4B Residuals Fitted to a Normal Distribution

TABLE II

AUTOCORRELATION TEST RESULTS

Data Set	Durbin-Watson Statistic	Degrees of Freedom	d_L, d_U (5% Significance Level)
all job orders combined	0.373	108	1.592,1.758(approx)
Job Order 687	0.190	34	1.208,1.728
Job Order 692	0.163	30	1.143,1.739
Job Order 701	0.242	29	1.124,1.743
Job Order 713	0.521	22	0.958,1.797
Job Order 720	0.436	25	1.038,1.767
Job Order 726	1.643	27	1.084,1.753
Job Order 731	0.219	26	1.062,1.759
Job Order 736	0.368	27	1.084,1.753
Job Order 746	0.944	13	0.574,2.094

VI. CONCLUSION

The objective of the research conducted by Womer and Gulledge was to incorporate economic theory into an airframe cost estimation model that is estimated from historical data and is sensitive to exogenous production schedule changes. Examination of the Womer-Gulledge model using F-4B cost data indicates their approach to be a valid and promising one. The results attained with their model are certainly comparable to those of other cost estimation models, and, at the same time, consistent with economic theory that other models have ignored. This conformity with accepted economic theory greatly enhances the model's usefulness to Navy management in planning and funding its aircraft acquisitions. The model has demonstrated a sensitivity to production rate changes that should enable a program manager to predict the effects of production schedule changes and to formulate and implement the most cost effective production schedule available.

Of particular significance in evaluating the model's potential applications is the consistency of its parameters across quite different airframe production programs. The parameter estimates reported in this thesis are quite similar to those attained by Womer and Gulledge in their studies of the C-141, F-102, and F-5/T-38 [Ref. 16]. By modifying β_0, the scaling parameter, to a level appropriate for the airframe of interest, the model can be adapted to plan the time path of resource usage for a new airframe before any cost data have been generated for the program. As actual production cost data become available, the preliminary cost estimates can be updated and refined using a Bayesian updating technique described by Womer and Gulledge [Ref. 16].

While the model does perform quite well and has readily apparent applications, it is not without flaws. One flaw is that the model allows only limited interaction among airframes. By assuming that all airframes in a job order are released simultaneously, the model does not allow work on an airframe to start later than the lot release date. This may create an artificially high production rate and understate the effects of learning over time. A second flaw is that the model includes no cost penalty for the hiring or firing of labor. Therefore, the model may predict that the work force should rise, then decline, and then rise again when hiring/firing costs may render a more moderate sized work force more cost effective during the relatively brief peaks and slumps in labor requirements. Another flaw of the model is the absence of any mechanism to adjust for the autocorrelation noted in Chapter V. The model should be modified to compensate for the temporal relationship in the data.

While not a flaw of the model itself, a potential problem in its implementation is data collection. The data typically collected is cumulative labor hours per lot of airframes. Without prior planning, it may be difficult to acquire the data to estimate the model accurately since the model calls for cost information on a per airframe basis. In this study, the model was modified slightly with unknown effects on accuracy since the data were aggregated on a job order level.

Despite these imperfections, the model should be considered a valuable tool for the decisionmaker. Owing to the intricacies and inexactness of the science of cost estimation, any and all such tools should be used to improve the decisionmaker's awareness of the factors that can affect a program's costs. As production scheduling does matter as a determinant of program cost, the model may enable decisionmakers to better understand the implications of funding cuts, stretchouts, and altered delivery schedules and, therefore, to make better decisions.

APPENDIX: F-4B DATA BASE

Data for each of the nine F-4B job orders are listed with one line of data entered for each month during which work occurred on a job order. Each line contains

L T2 D W N H T1

where L = labor (in 100,000 hours) for the job order this month

T2 = end of time period

D = # of airframes delivered prior to this job order

W = time period when work began on this job order

N = sequence # of the last airframe in the job order

H = average # of airframes in house during this period

T1 = start of time period

After these lines, the delivery dates for each of the airframes in the job order are listed.

JOB ORDER 687

0.23904	28.0	0.0	27.0	16.0	16.000	27.0
0.26792	29.0	0.0	27.0	16.0	16.000	28.0
0.28405	30.0	0.0	27.0	16.0	16.000	29.0
0.28837	31.0	0.0	27.0	16.0	16.000	30.0
0.33922	32.0	0.0	27.0	16.0	16.000	31.0
0.37202	33.0	0.0	27.0	16.0	16.000	32.0
0.61544	34.0	0.0	27.0	16.0	16.000	33.0
0.64844	35.0	0.0	27.0	16.0	16.000	34.0
0.88443	36.0	0.0	27.0	16.0	16.000	35.0
1.30476	37.0	0.0	27.0	16.0	16.000	36.0
1.90464	38.0	0.0	27.0	16.0	16.000	37.0
1.96346	39.0	0.0	27.0	16.0	40.000	38.0
1.94893	40.0	0.0	27.0	16.0	40.000	39.0
2.37299	41.0	0.0	27.0	16.0	40.000	40.0
2.21135	42.0	0.0	27.0	16.0	40.000	41.0
1.90135	43.0	0.0	27.0	16.0	40.000	42.0
2.00296	44.0	0.0	27.0	16.0	39.500	43.0
2.02364	45.0	0.0	27.0	16.0	38.500	44.0
1.89796	46.0	0.0	27.0	16.0	37.500	45.0
1.76414	47.0	0.0	27.0	16.0	108.000	46.0
1.82274	48.0	0.0	27.0	16.0	105.500	47.0
1.58376	49.0	0.0	27.0	16.0	102.500	48.0
1.49733	50.0	0.0	27.0	16.0	101.000	49.0
1.39265	51.0	0.0	27.0	16.0	101.000	50.0
1.36656	52.0	0.0	27.0	16.0	101.000	51.0
1.29265	53.0	0.0	27.0	16.0	101.000	52.0
0.75652	54.0	0.0	27.0	16.0	101.000	53.0
0.36923	55.0	0.0	27.0	16.0	99.500	54.0
0.11657	56.0	0.0	27.0	16.0	96.500	55.0
0.09666	57.0	0.0	27.0	16.0	94.500	56.0
0.06745	58.0	0.0	27.0	16.0	94.000	57.0
0.03575	59.0	0.0	27.0	16.0	94.000	58.0
0.04195	60.0	0.0	27.0	16.0	166.000	59.0
0.02468	61.0	0.0	27.0	16.0	165.000	60.0

DELIVERY DATES: 43.50 44.50 45.50 46.33 46.67 47.25 47.50
47.75 48.25 48.50 48.75 54.25 54.50 54.75 56.50 60.50

JOB ORDER 692

0.01967	39.0	16.0	38.0	40.0	40.000	38.0
0.10990	40.0	16.0	38.0	40.0	40.000	39.0
0.42763	41.0	16.0	38.0	40.0	40.000	40.0
0.51955	42.0	16.0	38.0	40.0	40.000	41.0
0.61842	43.0	16.0	38.0	40.0	40.000	42.0
0.77169	44.0	16.0	38.0	40.0	39.500	43.0
0.96280	45.0	16.0	38.0	40.0	38.500	44.0
1.05265	46.0	16.0	38.0	40.0	37.500	45.0
1.18733	47.0	16.0	38.0	40.0	108.000	46.0
1.58118	48.0	16.0	38.0	40.0	105.500	47.0
1.80794	49.0	16.0	38.0	40.0	102.500	48.0
2.22878	50.0	16.0	38.0	40.0	101.000	49.0
2.35663	51.0	16.0	38.0	40.0	101.000	50.0
2.71505	52.0	16.0	38.0	40.0	101.000	51.0
3.47529	53 0	16.0	38.0	40.0	101.000	52.0
2.59482	54.0	16.0	38.0	40.0	101.000	53.0
2.91105	55.0	16.0	38.0	40.0	99.500	54.0
2.70636	56.0	16.0	38.0	40.0	96.500	55.0
2.11521	57.0	16.0	38.0	40.0	94.500	56.0
1.90967	58.0	16.0	38.0	40.0	94.000	57.0
1.36859	59.0	16.0	38.0	40.0	94.000	58.0
1.42259	60.0	16.0	38.0	40.0	166.000	59.0
1.45556	61.0	16.0	38.0	40.0	165.000	60.0
1.12204	62.0	16.0	38.0	40.0	163.500	61.0
0.70945	63.0	16.0	38.0	40.0	162.500	62.0
0.23030	64.0	16.0	38.0	40.0	161.000	63.0
0.17782	65.0	16.0	38.0	40.0	156.500	64.0
0.07716	66.0	16.0	38.0	40.0	150.500	65.0
0.03211	67.0	16.0	38.0	40.0	142.500	66.0
0.00000	68.0	16.0	38.0	40.0	131.500	67.0

DELIVERY DATES: 55.25 55.50 55.75 60.50 61.50 62.50 63.33
63.67 64.13 64.25 64.38 64.50 64.63 64.75 64.88 65.17 65.33
65.50 65.66 65.83 66.25 66.50 66.75 67.50

JOB ORDER 701

0.00996	47.0	40.0	46.0	112.0	108.000	46.0
0.02965	48.0	40.0	46.0	112.0	105.500	47.0
0.08944	49.0	40.0	46.0	112.0	102.500	48.0
0.30266	50.0	40.0	46.0	112.0	101.000	49.0
0.63845	51.0	40.0	46.0	112.0	101.000	50.0
1.00236	52.0	40.0	46.0	112.0	101.000	51.0
1.40825	53.0	40.0	46.0	112.0	101.500	52.0
1.62892	54.0	40.0	46.0	112.0	101.000	53.0
2.03716	55.0	40.0	46.0	112.0	99.500	54.0
3.17816	56.0	40.0	46.0	112.0	96.500	55.0
3.62862	57.0	40.0	46.0	112.0	94.500	56.0
4.72402	58.0	40.0	46.0	112.0	94.000	57.0
4.78499	59.0	40.0	46.0	112.0	94.000	58.0
4.86147	60.0	40.0	46.0	112.0	166.000	59.0
5.34870	61.0	40.0	46.0	112.0	165.000	60.0
5.96560	62.0	40.0	46.0	112.0	163.500	61.0
6.34085	63.0	40.0	46.0	112.0	162.500	62.0
5.85055	64.0	40.0	46.0	112.0	161.000	63.0
6.24051	65.0	40.0	46.0	112.0	156.500	64.0
4.91988	66.0	40.0	46.0	112.0	150.500	65.0
4.38904	67.0	40.0	46.0	112.0	142.500	66.0
3.07483	68.0	40.0	46.0	112.0	131.500	67.0
2.07644	69.0	40.0	46.0	112.0	121.000	68.0
1.87063	70.0	40.0	46.0	112.0	229.000	69.0
0.99034	71.0	40.0	46.0	112.0	219.000	70.0
0.80919	72.0	40.0	46.0	112.0	211.000	71.0
0.31633	73.0	40.0	46.0	112.0	205.000	72.0
0.08406	74.0	40.0	46.0	112.0	199.000	73.0
0.05670	75.0	40.0	46.0	112.0	191.500	74.0

DELIVERY DATES: 66.11 66.22 66.33 66.44 66.55 66.66 66.77
66.88 67.09 67.18 67.27 67.36 67.45 67.55 67.64 67.73 67.82
67.91 68.09 68.18 68.27 68.36 68.45 68.55 68.64 68.73 68.82
68.91 69.09 69.18 69.27 69.36 69.45 69.55 69.64 69.73 69.82
69.91 70.09 70.18 70.27 70.36 70.45 70.55 70.64 70.73 70.82
70.91 71.14 71.29 71.43 71.57 71.71 71.86 72.14 72.29 72.43
72.57 72.71 72.86 73.14 73.29 73.43 73.57 73.71 73.86 74.14
74.29 74.43 74.57 74.71 74.86

0.09440	60.0	112.0	59.0	184.0	166.000	59.0
0.01798	61.0	112.0	59.0	184.0	165.000	60.0
0.50545	62.0	112.0	59.0	184.0	163.500	61.0
0.72456	63.0	112.0	59.0	184.0	162.500	62.0
1.01639	64.0	112.0	59.0	184.0	161.000	63.0
1.61539	65.0	112.0	59.0	184.0	156.500	64.0
1.86886	66.0	112.0	59.0	184.0	150.500	65.0
2.41520	67.0	112.0	59.0	184.0	142.500	66.0
2.61123	68.0	112.0	59.0	184.0	131.500	67.0
2.96616	69.0	112.0	59.0	184.0	121.000	68.0
4.75018	70.0	112.0	59.0	184.0	229.000	69.0
5.71782	71.0	112.0	59.0	184.0	219.000	70.0
6.07048	72.0	112.0	59.0	184.0	211.000	71.0
5.99359	73.0	112.0	59.0	184.0	205.000	72.0
5.38045	74.0	112.0	59.0	184.0	199.000	73.0
5.28531	75.0	112.0	59.0	184.0	191.500	74.0
3.12728	76.0	112.0	59.0	184.0	180.500	75.0
1.72110	77.0	112.0	59.0	184.0	167.000	76.0
0.94430	78.0	112.0	59.0	184.0	154.500	77.0
0.39982	79.0	112.0	59.0	184.0	145.500	78.0
0.17868	80.0	112.0	59.0	184.0	133.500	79.0
0.04702	81.0	112.0	59.0	184.0	120.000	80.0

DELIVERY DATES: 74.25 74.50 74.75 75.07 75.14 75.21 75.29
75.36 75.43 75.50 75.57 75.64 75.71 75.79 75.86 75.93 76.07
76.13 76.20 76.27 76.33 76.40 76.47 76.53 76.60 76.67 76.73
76.80 76.87 76.93 77.08 77.17 77.25 77.33 77.42 77.50 77.58
77.67 77.75 77.83 77.92 78.12 78.25 78.37 78.50 78.62 78.75
78.87 79.06 79.12 79.18 79.24 79.29 79.35 79.41 79.47 79.53
79.59 79.65 79.71 79.76 79.82 79.88 79.94 80.11 80.22 80.33
80.44 80.55 80.67 80.78 80.89

JOB ORDER 720

0.03207	70.0	184.0	69.0	302.0	229.000	69.0
0.34985	71.0	184.0	69.0	302.0	219.000	70.0
0.90954	72.0	184.0	69.0	302.0	211.000	71.0
1.76705	73.0	184.0	69.0	302.0	205.000	72.0
2.72706	74.0	184.0	69.0	302.0	199.000	73.0
3.85908	75.0	184.0	69.0	302.0	191.500	74.0
5.12503	76.0	184.0	69.0	302.0	180.500	75.0
6.94428	77.0	184.0	69.0	302.0	167.000	76.0
6.83042	78.0	184.0	69.0	302.0	154.500	77.0
6.08640	79.0	184.0	69.0	302.0	145.500	78.0
7.22808	80.0	184.0	69.0	302.0	133.500	79.0
5.19000	81.0	184.0	69.0	302.0	120.000	80.0
5.13461	82.0	184.0	69.0	302.0	110.500	81.0
5.66094	83.0	184.0	69.0	302.0	101.500	82.0
4.23694	84.0	184.0	69.0	302.0	212.500	83.0
2.90815	85.0	184.0	69.0	302.0	197.500	84.0
2.12347	86.0	184.0	69.0	302.0	184.000	85.0
0.98820	87.0	184.0	69.0	302.0	173.000	86.0
0.59670	88.0	184.0	69.0	302.0	164.500	87.0
0.31320	89.0	184.0	69.0	302.0	157.500	88.0
0.15440	90.0	184.0	69.0	302.0	150.500	89.0
0.10312	91.0	184.0	69.0	302.0	144.000	90.0
0.08971	92.0	184.0	69.0	302.0	138.000	91.0
0.06859	93.0	184.0	69.0	302.0	131.000	92.0
0.15482	94.0	184.0	69.0	302.0	122.500	93.0

DELIVERY DATES: 79.50 80.33 80.67 81.10 81.20 81.30 81.40
81.50 81.60 81.70 81.80 81.90 82.10 82.20 82.30 82.40 82.50
82.60 82.70 82.80 82.90 83.06 83.13 83.19 83.25 83.31 83.38
83.44 83.50 83.56 83.63 83.69 83.75 83.82 83.88 83.94 84.06
84.13 84.19 84.25 84.31 84.38 84.44 84.50 84.56 84.63 84.69
84.75 84.82 84.88 84.94 85.08 85.15 85.23 85.31 85.39 85.46
85.54 85.62 85.69 85.77 85.85 85.92 86.09 86.18 86.27 86.36
86.45 86.55 86.64 86.73 86.82 86.91 87.12 87.24 87.36 87.48
87.60 87.72 87.84 88.12 88.24 88.36 88.48 88.60 88.72 88.84
89.12 89.24 89.36 89.48 89.60 89.72 89.84 90.14 90.28 90.42
90.56 90.70 90.84 91.14 91.28 91.42 91.56 91.70 91.84 92.11
92.22 92.33 92.44 92.55 92.66 92.77 92.88 93.20 93.40 93.60
93.80

JOB ORDER 726

0.79894	84.0	302.0	83.0	425.0	212.500	83.0
1.44939	85.0	302.0	83.0	425.0	197.500	84.0
1.29627	86.0	302.0	83.0	425.0	184.000	85.0
1.14269	87.0	302.0	83.0	425.0	173.000	86.0
1.70337	88.0	302.0	83.0	425.0	164.500	87.0
2.93063	89.0	302.0	83.0	425.0	157.500	88.0
2.13076	90.0	302.0	83.0	425.0	150.500	89.0
2.88053	91.0	302.0	83.0	425.0	144.000	90.0
3.01235	92.0	302.0	83.0	425.0	138.000	91.0
2.66183	93.0	302.0	83.0	425.0	131.000	92.0
3.81447	94.0	302.0	83.0	425.0	122.500	93.0
3.09190	95.0	302.0	83.0	425.0	114.000	94.0
3.14733	96.0	302.0	83.0	425.0	106.000	95.0
3.88340	97.0	302.0	83.0	425.0	98.000	96.0
2.74050	98.0	302.0	83.0	425.0	216.500	97.0
2.50506	99.0	302.0	83.0	425.0	210.000	98.0
3.05831	100.0	302.0	83.0	425.0	202.000	99.0
2.10908	101.0	302.0	83.0	425.0	194.000	100.0
1.71385	102.0	302.0	83.0	425.0	185.500	101.0
1.86316	103.0	302.0	83.0	425.0	177.000	102.0
1.33472	104.0	302.0	83.0	425.0	168.500	103.0
1.09528	105.0	302.0	83.0	425.0	159.500	104.0
0.74063	106.0	302.0	83.0	425.0	151.000	105.0
0.44760	107.0	302.0	83.0	425.0	143.000	106.0
0.37475	108.0	302.0	83.0	425.0	241.000	107.0
0.64395	109.0	302.0	83.0	425.0	234.500	108.0
0.13949	110.0	302.0	83.0	425.0	228.000	109.0

DELIVERY DATES: 93.16 93.32 93.48 93.64 93.80 94.11 94.22
94.33 94.44 94.55 94.66 94.77 94.88 95.11 95.22 95.33 95.44
95.55 95.66 95.77 95.88 96.11 96.22 96.33 96.44 96.55 96.66
96.77 96.88 97.16 97.32 97.48 97.64 97.80 98.11 98.22 98.33
98.44 98.55 98.66 98.77 98.88 99.11 99.22 99.33 99.44 99.55
99.66 99.77 99.88 100.11 100.22 100.33 100.44 100.55 100.66
100.77 100.88 101.10 101.20 101.30 101.40 101.50 101.60
101.70 101.80 101.90 102.11 102.22 102.33 102.44 102.55
102.66 102.77 102.88 103.10 103.20 103.30 103.40 103.50
103.60 103.70 103.80 103.90 104.10 104.20 104.30 104.40
104.50 104.60 104.70 104.80 104.90 105.11 105.22 105.33
105.44 105.55 105.66 105.77 105.88 106.11 106.22 106.33
106.44 106.55 106.66 106.77 106.88 107.11 107.22 107.33
107.44 107.55 107.66 107.77 107.88 108.16 108.32 108.48
108.64 108.80 109.50

JOB ORDER 731

0.22886	98.0	425.0	97.0	550.0	216.500	97.0
0.36007	99.0	425.0	97.0	550.0	210.000	98.0
0.67462	100.0	425.0	97.0	550.0	202.000	99.0
0.68518	101.0	425.0	97.0	550.0	194.000	100.0
0.93599	102.0	425.0	97.0	550.0	185.500	101.0
1.24208	103.0	425.0	97.0	550.0	177.000	102.0
1.24028	104.0	425.0	97.0	550.0	168.500	103.0
1.79210	105.0	425.0	97.0	550.0	159.500	104.0
1.94067	106.0	425.0	97.0	550.0	151.000	105.0
2.20895	107.0	425.0	97.0	550.0	143.000	106.0
3.13766	108.0	425.0	97.0	550.0	241.000	107.0
2.05092	109.0	425.0	97.0	550.0	234.500	108.0
2.44626	110.0	425.0	97.0	550.0	228.000	109.0
3.01853	111.0	425.0	97.0	550.0	219.000	110.0
2.31921	112.0	425.0	97.0	550.0	208.500	111.0
2.24526	113.0	425.0	97.0	550.0	198.500	112.0
2.46908	114.0	425.0	97.0	550.0	187.500	113.0
1.70196	115.0	425.0	97.0	550.0	176.000	114.0
1.51614	116.0	425.0	97.0	550.0	166.000	115.0
1.59310	117.0	425.0	97.0	550.0	156.000	116.0
0.97895	118.0	425.0	97.0	550.0	145.500	117.0
0.65454	119.0	425.0	97.0	550.0	135.000	118.0
0.56889	120.0	425.0	97.0	550.0	125.000	119.0
0.16873	121.0	425.0	97.0	550.0	118.000	120.0
0.17997	122.0	425.0	97.0	550.0	111.500	121.0
0.05002	123.0	425.0	97.0	550.0	106.500	122.0

DELIVERY DATES: 109.12 109.25 109.37 109.50 109.62 109.75
109.87 110.09 110.18 110.27 110.36 110.45 110.54 110.63
110.72 110.81 110.90 111.08 111.16 111.24 111.32 111.40
111.48 111.56 111.64 111.72 111.80 111.88 112.10 112.20
112.30 112.40 112.50 112.60 112.70 112.80 112.90 113.07
113.14 113.21 113.28 113.35 113.42 113.49 113.56 113.63
113.70 113.77 113.84 113.91 114.09 114.18 114.27 114.36
114.45 114.54 114.63 114.72 114.81 114.90 115.09 115.18
115.27 115.36 115.45 115.54 115.63 115.72 115.81 115.90
116.09 116.18 116.27 116.36 116.45 116.54 116.63 116.72
116.81 116.90 117.16 117.24 117.32 117.40 117.48 117.56
117.64 117.72 117.80 117.88 118.09 118.18 118.27 118.36
118.45 118.54 118.63 118.72 118.81 118.90 119.09 119.18
119.27 119.36 119.45 119.54 119.63 119.72 119.81 119.90
120.20 120.40 120.60 120.80 121.10 121.20 121.30 121.40
121.50 121.60 121.70 121.80 121.90 122.50

JOB ORDER 736

0.00429	108.0	550.0	107.0	656.0	241.000	107.0
0.01981	109.0	550.0	107.0	656.0	234.500	108.0
0.07112	110.0	550.0	107.0	656.0	228.000	109.0
0.18193	111.0	550.0	107.0	656.0	219.000	110.0
0.22616	112.0	550.0	107.0	656.0	208.500	111.0
0.36635	113.0	550.0	107.0	656.0	198.500	112.0
0.62820	114.0	550.0	107.0	656.0	187.500	113.0
0.66860	115.0	550.0	107.0	656.0	176.000	114.0
0.75417	116.0	550.0	107.0	656.0	166.000	115.0
1.26485	117.0	550.0	107.0	656.0	156.000	116.0
1.37495	118.0	550.0	107.0	656.0	145.500	117.0
1.53253	119.0	550.0	107.0	656.0	135.000	118.0
2.31180	120.0	550.0	107.0	656.0	125.000	119.0
1.11954	121.0	550.0	107.0	656.0	118.000	120.0
2.33234	122.0	550.0	107.0	656.0	111.500	121.0
3.56218	123.0	550.0	107.0	656.0	106.500	122.0
2.88370	124.0	550.0	107.0	656.0	96.500	123.0
2.60084	125.0	550.0	107.0	656.0	86.000	124.0
2.86669	126.0	550.0	107.0	656.0	75.000	125.0
2.03427	127.0	550.0	107.0	656.0	61.500	126.0
1.72168	128.0	550.0	107.0	656.0	47.500	127.0
1.63850	129.0	550.0	107.0	656.0	36.500	128.0
0.97515	130.0	550.0	107.0	656.0	26.500	129.0
0.44764	131.0	550.0	107.0	656.0	20.500	130.0
0.47514	132.0	550.0	107.0	656.0	15.000	131.0
0.41039	133.0	550.0	107.0	656.0	8.500	132.0
0.32228	134.0	550.0	107.0	656.0	3.500	133.0

DELIVERY DATES: 122.11 122.22 122.33 122.44 122.55 122.66
122.77 122.88 123.08 123.16 123.24 123.32 123.40 123.48
123.56 123.64 123.72 123.80 123.88 124.09 124.18 124.27
124.36 124.45 124.54 124.63 124.72 124.81 124.90 125.07
125.15 125.22 125.30 125.37 125.45 125.52 125.60 125.67
125.75 125.82 125.90 126.06 126.12 126.18 126.24 126.30
126.36 126.42 126.48 126.54 126.60 126.66 126.72 126.78
126.84 126.90 127.07 127.14 127.21 127.28 127.35 127.42
127.49 127.56 127.63 127.70 127.77 127.84 127.91 128.10
128.20 128.30 128.40 128.50 128.60 128.70 128.80 128.90
129.10 129.20 129.30 129.40 129.50 129.60 129.70 129.80
129.90 130.26 130.32 130.48 130.64 130.80 131.14 131.28
131.41 131.56 131.70 131.84 132.14 132.28 132.42 132.56
132.70 132.84 133.33 133.67

JOB ORDER 746

0.09572	123.0	656.0	122.0	660.0	106.500	122.0
0.04738	124.0	656.0	122.0	660.0	96.500	123.0
0.05494	125.0	656.0	122.0	660.0	86.000	124.0
0.08816	126.0	656.0	122.0	660.0	75.000	125.0
0.11121	127.0	656.0	122.0	660.0	61.500	126.0
0.14372	128.0	656.0	122.0	660.0	47.500	127.0
0.20073	129.0	656.0	122.0	660.0	36.500	128.0
0.14884	130.0	656.0	122.0	660.0	26.500	129.0
0.08745	131.0	656.0	122.0	660.0	20.500	130.0
0.06917	132.0	656.0	122.0	660.0	15.000	131.0
0.04802	133.0	656.0	122.0	660.0	8.500	132.0
0.02105	134.0	656.0	122.0	660.0	3.500	133.0
0.04414	135.0	656.0	122.0	660.0	1.000	134.0

DELIVERY DATES: 132.50 133.50 134.33 134.67

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